Effect of Fock terms on nuclear symmetry energy based on Lorentz-covariant decomposition of nucleon self-energies

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# Table of contents

1. Introduction
2. Lorentz-covariant decomposition of nuclear symmetry energy
3. Relativistic mean-field Lagrangian density
4. Numerical results
5. Summary
Introduction

The nuclear symmetry energy, $E_{\text{sym}}$, is known to be an important physical quantity not only in nuclear physics but also in astrophysics.

Massive neutron stars
Gravitational wave

EoS for neutron stars $\Rightarrow E_{\text{sym}}$ at extremely high densities

To study the density-dependence of $E_{\text{sym}}$

Motivation:

- Using the relativistic Hartree-Fock (RHF) approximation, we study the effect of the Fock terms on $E_{\text{sym}}$ not only around the saturation density but also at higher densities.
- By taking into account the Lorentz-covariant decomposition of the nucleon self-energy, we investigate how the momentum dependence due to the exchange contribution affects $E_{\text{sym}}$.

$\Rightarrow$ We compare the theoretical results with the experimental data (FFG model).
Nucleon self-energies

The nucleon self-energy is given by the Lorentz-covariant form with scalar ($s$), time ($0$), and space ($v$) components.

$$\Sigma_N(k) = \Sigma^s_N(k) - \gamma_0 \Sigma^0_N(k) + \left( \vec{\gamma} \cdot \vec{k} \right) \Sigma^v_N \quad (N = n, p).$$

The $\Sigma^{s,0,v}_N$ are generally composed of the direct and exchange diagrams.

$$\Sigma^i_N(k) = \Sigma^{i,\text{dir}}_N + \Sigma^{i,\text{ex}}_N, \quad i = s, 0, v.$$ 

Inserting this form into the Dirac equation, we get the effective nucleon mass and momentum in nuclear matter.

$$M^*_N(k) = M_N + \Sigma^s_N(k), \quad k^*_N(k) = \left( k^0 + \Sigma^0_N(k), \vec{k} + \vec{k} \Sigma^v_N(k) \right).$$
Lorentz-covariant decomposition of $E_{\text{sym}}$

According to the Hugenholtz–Van Hove (HVH) theorem, the nucleon chemical potential in asymmetric nuclear matter should be equal to its Fermi energy. Thus, at zero temperature, the single-particle energy at Fermi surface is generally given by

$$E_N(k_{FN}) = \frac{d(\rho_B E_B)}{d\rho_B},$$

where the Fermi momentum for nucleon, $k_{FN}$, reads $k_{FN} = (3\pi^2 \rho_N)^{1/3}$ ($N = n$ or $p$). Therefore, $E_{\text{sym}}$ can be written as

$$E_{\text{sym}} = \frac{1}{8} \rho_B \left( \frac{\partial}{\partial \rho_p} - \frac{\partial}{\partial \rho_n} \right) \left[ E_p(k_{Fp}) - E_n(k_{Fn}) \right]_{\rho_p = \rho_n}.$$

In the relativistic mean-field (RMF) models, $E_{\text{sym}}$ is divided into the kinetic and potential parts as

$$E_{\text{sym}} = E_{\text{sym}}^{\text{kin}} + E_{\text{sym}}^{\text{pot}}$$

$$= E_{\text{sym}}^{\text{kin}} + E_{\text{sym}}^{s} + E_{\text{sym}}^{0} + E_{\text{sym}}^{v} = \frac{1}{6} k_F^* E_F^* k_F + \frac{1}{8} \rho_B \left( \frac{M_N^*}{E_F^*} \partial \Sigma_{\text{sym}}^s - \partial \Sigma_{\text{sym}}^0 + \frac{k_F^*}{E_F^*} \partial \Sigma_{\text{sym}}^v \right),$$

with $k_F = k_{Fp} = k_{Fn}$, $E_F^* = \sqrt{k_F^* + M_N^*}$, and $\partial \Sigma_{\text{sym}}^{s(0)[v]} \equiv \left( \frac{\partial}{\partial \rho_p} - \frac{\partial}{\partial \rho_n} \right) \left( \Sigma_{p}^{s(0)[v]} - \Sigma_{n}^{s(0)[v]} \right)_{\rho_p = \rho_n}$. 

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with \( k_{F} = k_{Fp} = k_{Fn}, \ E_{F}^{*} = \sqrt{k_{F}^{*2} + M_N^{*2}}, \) and \( \partial \Sigma_{s}^{s}[v] \equiv \left( \frac{\partial}{\partial \rho_p} - \frac{\partial}{\partial \rho_n} \right) \left( \Sigma_{p}^{s}(0)[v] - \Sigma_{n}^{s}(0)[v] \right)_{\rho_p=\rho_n}. \)
Relativistic mean-field (RMF) Lagrangian density for uniform hadronic matter:

\[ \mathcal{L} = \sum_{N=p,n} \bar{\psi}_N \left( i \gamma_\mu \partial^\mu - M_N \right) \psi_N + \mathcal{L}_M + \mathcal{L}_{\text{int}} - U_{\text{NL}}. \]

Interaction Lagrangian density: mesons (\( \sigma, \omega, \vec{\pi}, \) and \( \vec{\rho} \))

\[ \mathcal{L}_{\text{int}} = \sum_{N=p,n} \left( \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\pi + \mathcal{L}_\rho \right). \]

- \( \mathcal{L}_\sigma = g_{\sigma N} \bar{\psi}_N \sigma \psi_N \), scalar
- \( \mathcal{L}_\omega = -g_{\omega N} \bar{\psi}_N \gamma_\mu \omega^\mu \psi_N \), vector
- \( \mathcal{L}_\pi = -\frac{f_{\pi N}}{m_\pi} \bar{\psi}_N \gamma_5 \gamma_\mu \partial^\mu \vec{\pi} \psi_N \cdot \vec{\tau}_N \), pseudovector
- \( \mathcal{L}_\rho = -g_{\rho N} \bar{\psi}_N \gamma_\mu \vec{\rho}^\mu \psi_N \cdot \vec{\tau}_N + \frac{f_{\rho N}}{2M} \bar{\psi}_N \sigma_{\mu \nu} \partial^\nu \vec{\rho}^\mu \psi_N \cdot \vec{\tau}_N \), vector
- \( \mathcal{L}_\rho = -g_{\rho N} \bar{\psi}_N \gamma_\mu \vec{\rho}^\mu \psi_N \cdot \vec{\tau}_N + \frac{f_{\rho N}}{2M} \bar{\psi}_N \sigma_{\mu \nu} \partial^\nu \vec{\rho}^\mu \psi_N \cdot \vec{\tau}_N \), tensor

The following nonlinear term is also introduced in order to reproduce the saturation properties of nuclear matter at the mean-field level:

\[ U_{\text{NL}} = \frac{1}{3} g_2 \bar{\sigma}^3 + \frac{1}{4} g_3 \bar{\sigma}^4. \]
**Nuclear symmetry energy at \( \rho_0 \)**

\[
U^\text{SEP}_N (k, \epsilon_k) = \sum^s_N (k) - \frac{E^N_N (k)}{M^N_N} \sum^0_N (k) + \frac{1}{2M^N_N} \left( \left[ \sum^s_N (k) \right]^2 - \left[ \sum^0_N (k) \right]^2 \right)
\]

- **RHF1**: CD-Bonn
- **RHF2**: adjusting cutoff parameters so as to fit \( U^\text{SEP}_N \)

\[
E_{\text{sym}} = E_{\text{sym}}^{\text{kin}} + E_{\text{sym}}^{\text{pot}} = E_{\text{sym}}^{\text{kin}} + E_{\text{sym}}^{s} + E_{\text{sym}}^{0} + E_{\text{sym}}^{v}
\]

**Summary**: Exchange contribution mainly affects \( E_{\text{sym}}^{\text{pot}} \)

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**Figures**

- Graph showing the dependence of \( U^\text{SEP}_N \) on \( \epsilon_k \) with data points from X.-H. Li et al. and Hama et al.
- Bar chart showing the components of \( E_{\text{sym}} \) with \( E_{\text{sym}} \approx 32.5 \) (fixed) and exchange contribution.

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Density dependence of nuclear symmetry energy

Lorentz-covariant decomposition of $E_{\text{sym}}^{\text{pot}}$:

$$E_{\text{sym}}(\rho_B) = E_{\text{sym}}^{\text{kin}}(\rho_B) + E_{\text{sym}}^{\text{pot}}(\rho_B)$$

$$= E_{\text{sym}}^{\text{kin}}(\rho_B) + E_{\text{sym}}^{s}(\rho_B) + E_{\text{sym}}^{0}(\rho_B) + E_{\text{sym}}^{v}(\rho_B).$$

The free Fermi gas (FFG) model:

$$E_{\text{sym}}(\rho_B) = E_{\text{sym}}^{\text{kin}}(\rho_0) \left( \frac{\rho_B}{\rho_0} \right)^{2/3} + E_{\text{sym}}^{\text{pot}}(\rho_0) \left( \frac{\rho_B}{\rho_0} \right)^\gamma.$$ 

Constraints:


$$E_{\text{sym}}^{\text{pot}}(\rho_0) \left( \frac{\rho_B}{\rho_0} \right)^\gamma \text{ with } \gamma = 0.7^{+0.35}_{-0.3} \text{ (FFG model),}$$

- **RH**: $\gamma = 1.00$
- **RHF1**: $\gamma = 0.74$
- **RHF2**: $\gamma = 1.09$
Meson contribution to potential part

Potential part of $E_{\text{sym}}$:

$$E_{\text{sym}}^{\text{pot}}(\rho_B) = E_{\text{sym}}^{s}(\rho_B) + E_{\text{sym}}^{0}(\rho_B) + E_{\text{sym}}^{v}(\rho_B).$$

- **Relativistic Hartree (RH) approximation:** only $\rho$ meson
  
  $$E_{\text{sym}}^{\text{pot}}(\rho_B) = E_{\text{sym}}^{0,\text{dir}}(\rho_B) = \frac{1}{2} \frac{g_{\rho}^2}{m_{\rho}^2} \rho_B.$$

- **Relativistic Hartree-Fock (RHF) approximation:**
  
  $$E_{\text{sym}}^{\text{pot}}(\rho_B) = E_{\text{sym}}^{0,\text{dir}}(\rho_B) + \sum_{i=s,0,v} E_{\text{sym}}^{i,\text{ex}}(\rho_B)$$
  
  $$= \frac{1}{2} \frac{g_{\rho}^2}{m_{\rho}^2} \rho_B + \sum_{i=s,0,v} \sum_{M=\sigma,\omega,\pi,\rho} E_{\text{sym},M}^{i,\text{ex}}(\rho_B).$$

Not only $\rho$ meson but also $\sigma$, $\omega$, and $\pi$ mesons give influence on $E_{\text{sym}}^{\text{pot}}$ through the exchange diagrams. In particular, the $\sigma$ and $\omega$ mesons play an important role in $E_{\text{sym}}^{\text{pot}}$. On the other hand, the contributions due to $\rho$ and $\pi$ mesons are extremely small even at high densities.
Summary

Motivation:

✓ Using the Lorentz-covariant decomposition of nucleon self-energies based on the HVH theorem, we have studied the effect of Fock terms on $E_{\text{sym}}$.

Results:

✓ Fock terms strongly affect the potential part, $E_{\text{sym}}^{\text{pot}}$.

✓ As a consequence, $E_{\text{sym}}^{\text{pot}}$ in the RHF1 case becomes consistent with the constraint from heavy-ion collision data with the ImQMD transport model.

✓ The $\sigma$ and $\omega$ mesons make a significant contribution to $E_{\text{sym}}^{\text{pot}}$.

✓ Therefore, it is very important to include the exchange diagrams for understanding dense matter physics, and more precise calculations with RHF approximation are required in the future.

Thank You for Your Attention.