Effect of Fock terms on nuclear symmetry energy based on Lorentz-covariant decomposition of nucleon self-energies

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Introduction

The nuclear symmetry energy, $E_{\rm sym}$, is known to be a important physical quantity not only **in nuclear physics** but also **in astrophysics**.

Massive neutron stars
Gravitational wave } EoS for neutron stars $\Longrightarrow$ $E_{\rm sym}$ at extremely high densities

To study the density-dependence of $E_{\rm sym}$

Motivation:

- Using **the relativistic Hartree-Fock (RHF)** approximation, we study the effect of **the Fock terms** on $E_{\rm sym}$ not only around the saturation density but also at higher densities.
- By taking into account **the Lorentz-covariant decomposition of the nucleon self-energy**, we investigate how the momentum dependence due to **the exchange contribution** affects $E_{\rm sym}$.

$\Rightarrow$ We compare the theoretical results with the experimental data (FFG model).

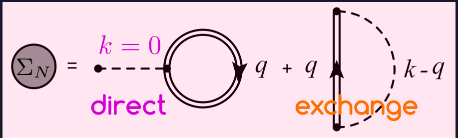
Nucleon self-energies

The nucleon self-energy is given by the Lorentz-covariant form with scalar (s), time (0), and space (v) components.

$$\Sigma_N(k) = \Sigma_N^s(k) - \gamma_0 \Sigma_N^0(k) + \left(\vec{\gamma} \cdot \hat{k}\right) \Sigma_N^v \quad (N = n, p).$$

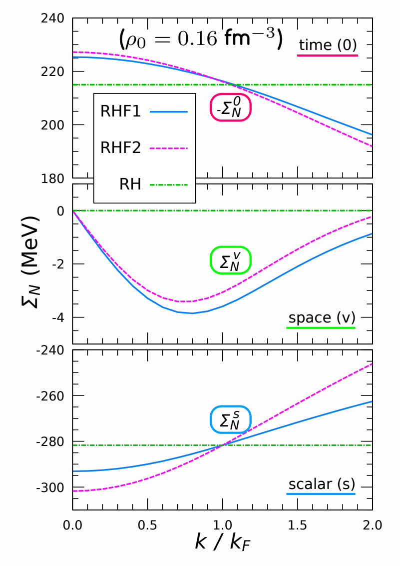
The $\Sigma_N^{s,0,v}$ are generally composed of the direct and exchange diagrams.

$$\Sigma_N^i(k) = \Sigma_N^{i,\mathrm{dir}} + \Sigma_N^{i,\mathrm{ex}}, \quad i = s, 0, v.$$



Inserting this form into the Dirac equation, we get the effective nucleon mass and momentum in nuclear matter.

$$M_N^*(k) = M_N + \Sigma_N^s(k), \quad k_N^*(k) = \left(k^0 + \Sigma_N^0(k), \vec{k} + \hat{k}\Sigma_N^v(k)\right).$$



Lorentz-covariant decomposition of $E_{\rm sym}$

According to the Hugenholtz–Van Hove (HVH) theorem, the nucleon chemical potential in asymmetric nuclear matter should be equal to its Fermi energy. Thus, at zero temperature, the single-particle energy at Fermi surface is generally given by

$$E_N(k_{F_N}) = \frac{d(\rho_B E_B)}{d\rho_B},$$

where the Fermi momentum for nucleon, k_{F_N}, reads $k_{F_N} = (3\pi^2\rho_N)^{1/3}$ ($N = n$ or p). Therefore, $E_{\rm sym}$ can be written as

$$E_{\rm sym} = \frac{1}{8}\rho_B \left(\frac{\partial}{\partial\rho_p} - \frac{\partial}{\partial\rho_n} \right) \left[E_p(k_{F_p}) - E_n(k_{F_n}) \right]_{\rho_p=\rho_n}.$$

In the relativistic mean-field (RMF) models, $E_{\rm sym}$ is divided into the kinetic and potential parts as

$$\begin{aligned} E_{\rm sym} &= E_{\rm sym}^{\bf kin} + E_{\rm sym}^{\bf pot} \\ &= E_{\rm sym}^{\bf kin} + E_{\rm sym}^{s} + E_{\rm sym}^{0} + E_{\rm sym}^{v} = \frac{1}{6}\frac{k_F^*}{E_F^*}k_F + \frac{1}{8}\rho_B \left(\frac{M_N^*}{E_F^*}\partial\Sigma_{\rm sym}^{s} - \partial\Sigma_{\rm sym}^{0} + \frac{k_F^*}{E_F^*}\partial\Sigma_{\rm sym}^{v} \right), \end{aligned}$$

with $k_F = k_{F_p} = k_{F_n}$, $E_F^* = \sqrt{k_F^{*2} + M_N^{*2}}$, and $\partial\Sigma_{\rm sym}^{s(0)[v]} \equiv \left(\frac{\partial}{\partial\rho_p} - \frac{\partial}{\partial\rho_n} \right) \left(\Sigma_p^{s(0)[v]} - \Sigma_n^{s(0)[v]} \right)_{\rho_p=\rho_n}$.

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Lorentz-covariant form of nucleon self-energies

$$\begin{aligned} E_{\rm sym} &= E_{\rm sym}^{\rm kin} + E_{\rm sym}^{\rm pot} \\ &= E_{\rm sym}^{\rm kin} + E_{\rm sym}^{s} + E_{\rm sym}^{0} + E_{\rm sym}^{v} = \frac{1}{6}\frac{k_F^*}{E_F^*}k_F + \frac{1}{8}\rho_B\left(\frac{M_N^*}{E_F^*}\partial\Sigma_{\rm sym}^s - \partial\Sigma_{\rm sym}^0 + \frac{k_F^*}{E_F^*}\partial\Sigma_{\rm sym}^v\right), \end{aligned}$$

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RMF Lagrangian density

Relativistic mean-field (RMF) Lagrangian density for uniform hadronic matter:

$$\mathcal{L} = \sum_{N=p,n} \bar{\psi}_N \left(i\gamma_\mu \partial^\mu - M_N \right) \psi_N + \mathcal{L}_M + \mathcal{L}_{\rm int} - U_{\rm NL}.$$

Interaction Lagrangian density: mesons (σ, ω, $\vec{\pi}$, and $\vec{\rho}$)

$$\mathcal{L}_{\rm int} = \sum_{N=p,n} \left(\mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\pi + \mathcal{L}_\rho \right),$$

$$\mathcal{L}_\sigma = g_{\sigma N} \bar{\psi}_N \sigma \psi_N, \qquad \mathcal{L}_\omega = -g_{\omega N} \bar{\psi}_N \gamma_\mu \omega^\mu \psi_N, \qquad \mathcal{L}_\pi = -\frac{f_{\pi N}}{m_\pi} \bar{\psi}_N \gamma_5 \gamma_\mu \partial^\mu \vec{\pi} \psi_N \cdot \vec{\tau}_N,$$

scalar vector pseudovector

$$\mathcal{L}_\rho = -g_{\rho N} \bar{\psi}_N \gamma_\mu \vec{\rho}^{\,\mu} \psi_N \cdot \vec{\tau}_N + \frac{f_{\rho N}}{2\mathcal{M}} \bar{\psi}_N \sigma_{\mu\nu} \partial^\nu \vec{\rho}^{\,\mu} \psi_N \cdot \vec{\tau}_N.$$

vector tensor

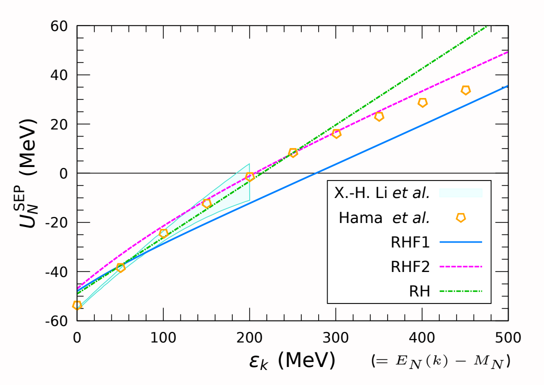
The following nonlinear term is also introduced in order to reproduce the saturation properties of nuclear matter at the mean-field level:

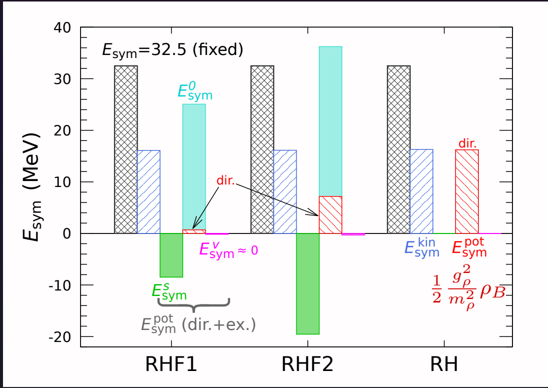
$$U_{\rm NL} = \frac{1}{3} g_2 \bar{\sigma}^3 + \frac{1}{4} g_3 \bar{\sigma}^4.$$

Nuclear symmetry energy at ρ_0

$$U_N^{\rm SEP}(k,\epsilon_k) = \Sigma_N^s(k) - \frac{E_N(k)}{M_N}\Sigma_N^0(k) + \frac{1}{2M_N}\left([\Sigma_N^s(k)]^2 - [\Sigma_N^0(k)]^2\right)$$

$$E_{\rm sym} = E_{\rm sym}^{\bf kin} + E_{\rm sym}^{\bf pot} = E_{\rm sym}^{\bf kin} + E_{\rm sym}^{s} + E_{\rm sym}^{0} + E_{\rm sym}^{v}$$





RHF1: CD-Bonn
RHF2: adjusting cutoff parameters so as to fit $U_N^{\rm SEP}$

exchange contribution mainly affects $E_{\rm sym}^{\rm pot}$

Density dependence of nuclear symmetry energy

Lorentz-covariant decomposition of $E_{\rm sym}^{\rm pot}$:

$$E_{\rm sym}(\rho_B) = E_{\rm sym}^{\rm kin}(\rho_B) + E_{\rm sym}^{\rm pot}(\rho_B)$$
$$= E_{\rm sym}^{\rm kin}(\rho_B) + E_{\rm sym}^{s}(\rho_B) + E_{\rm sym}^{0}(\rho_B) + E_{\rm sym}^{v}(\rho_B).$$

The free Fermi gas (FFG) model:

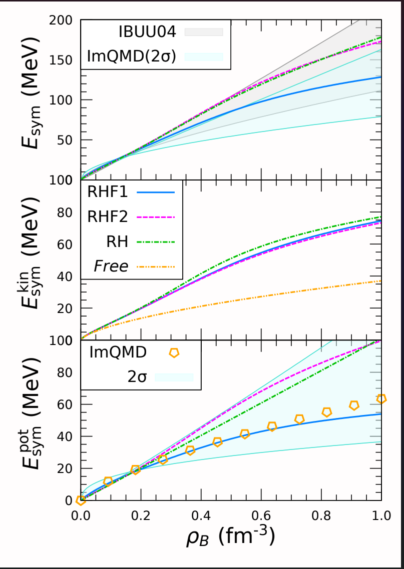
$$E_{\rm sym}(\rho_B) = E_{\rm sym}^{\rm kin}(\rho_0)\left(\frac{\rho_B}{\rho_0}\right)^{2/3} + E_{\rm sym}^{\rm pot}(\rho_0)\left(\frac{\rho_B}{\rho_0}\right)^{\gamma}.$$

Constraints:

- ImQMD: Improved quantum molecular dynamics transport model [M. B. Tsang, et al., Phys. Rev. Lett. **102**, 122701 (2009)].

$$E_{\rm sym}^{\rm pot}(\rho_0)\left(\frac{\rho_B}{\rho_0}\right)^{\gamma}$$ with $\gamma = 0.7^{+0.35}_{-0.3}$ (FFG model),

- RH: $\gamma = 1.00$
- RHF1: $\gamma = 0.74$
- RHF2: $\gamma = 1.09$



Meson contribution to potential part

Potential part of $E_{\rm sym}$:

$$E_{\rm sym}^{\rm pot}(\rho_B) = E_{\rm sym}^{s}(\rho_B) + E_{\rm sym}^{0}(\rho_B) + E_{\rm sym}^{v}(\rho_B).$$

- Relativistic Hartree (RH) approximation: only ρ meson

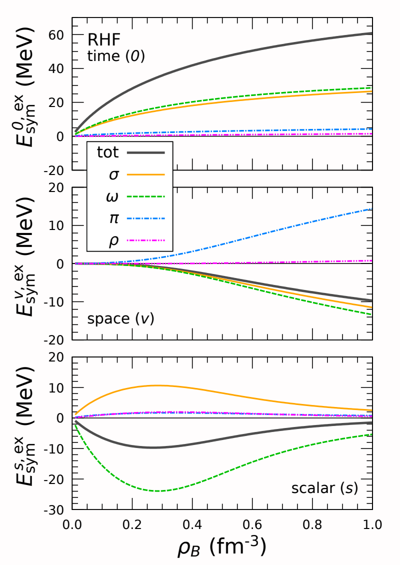
$$E_{\rm sym}^{\rm pot}(\rho_B) = E_{\rm sym}^{0,{\rm dir}}(\rho_B) = \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_B.$$

- Relativistic Hartree-Fock (RHF) approximation:

$$E_{\rm sym}^{\rm pot}(\rho_B) = E_{\rm sym}^{0,{\rm dir}}(\rho_B) + \sum_{i=s,0,v} E_{\rm sym}^{i,{\rm ex}}(\rho_B)$$

$$= \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_B + \sum_{i=s,0,v}\ \sum_{M=\sigma,\omega,\pi,\rho} E_{{\rm sym},M}^{i,{\rm ex}}(\rho_B).$$

Not only ρ meson but also σ, ω, and π mesons give influence on $E_{\rm sym}^{\rm pot}$ through the exchange diagrams.
In particular, the σ and ω mesons play a important role in $E_{\rm sym}^{\rm pot}$.
On the other hand, the contributions due to ρ and π mesons are extremely small even at high densities.



Summary

Motivation:

- ✓ Using the Lorentz-covariant decomposition of nucleon self-energies based on the HVH theorem, we have studied the effect of Fock terms on E_{sym}.

Results:

- ✓ Fock terms strongly affect the potential part, $E_{\mathrm{sym}}^{\mathrm{pot}}$.
- ✓ As a consequence, $E_{\mathrm{sym}}^{\mathrm{pot}}$ in the RHF1 case becomes consistent with the constraint from heavy-ion collision data with the ImQMD transport model.
- ✓ The σ and ω mesons make a significant contribution to $E_{\mathrm{sym}}^{\mathrm{pot}}$.
- ✓ Therefore, it is very important to include the exchange diagrams for understanding dense matter physics, and more precise calculations with RHF approximation are required in the future.

Thank You for Your Attention.