Study of Ξ-nucleus and Ξ-atom based on the ΞN interaction from QCD on lattice

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for

HAL QCD Collaboration

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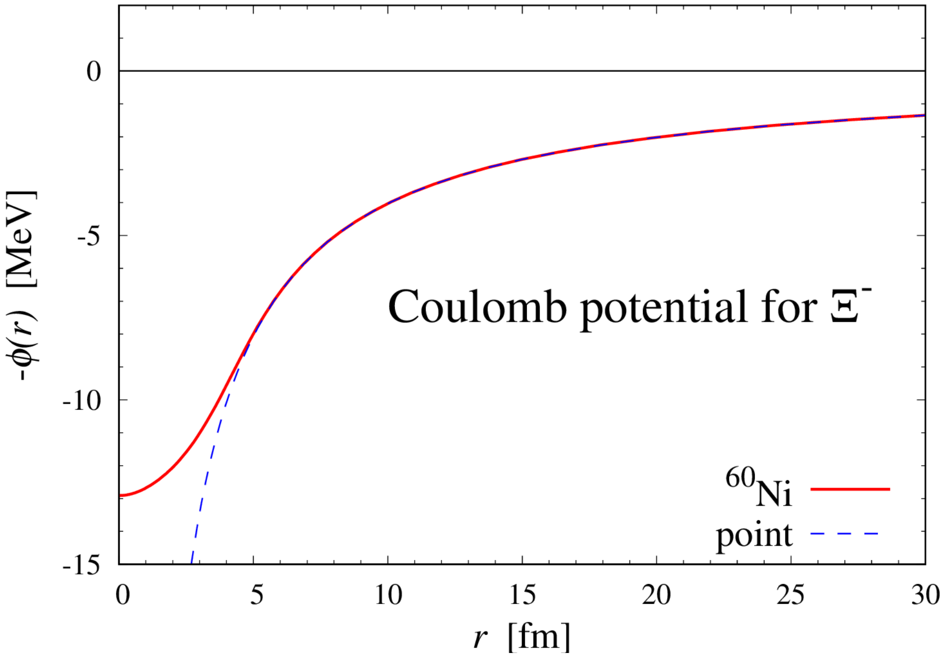
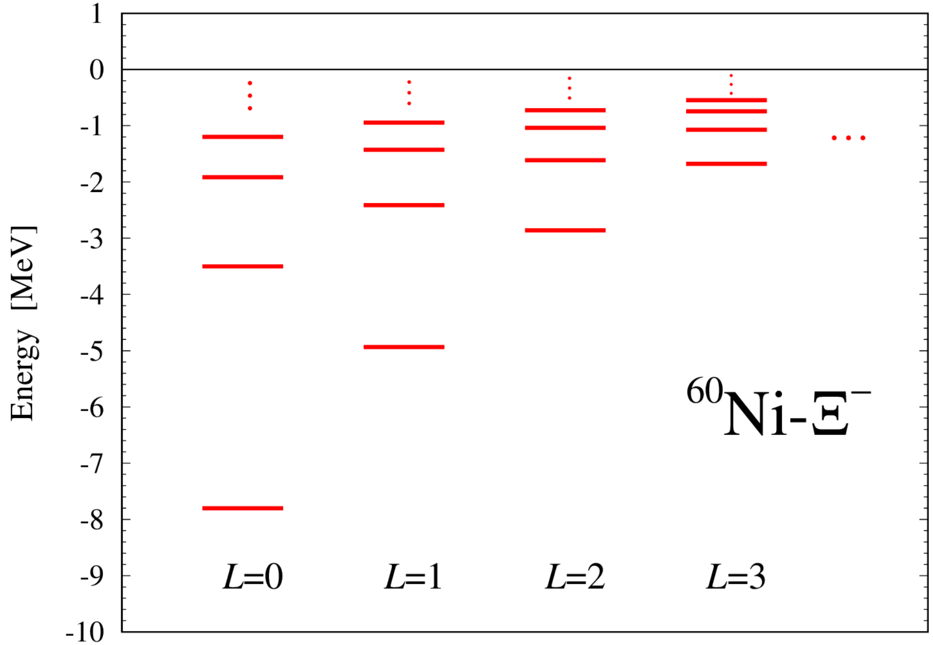
Introduction

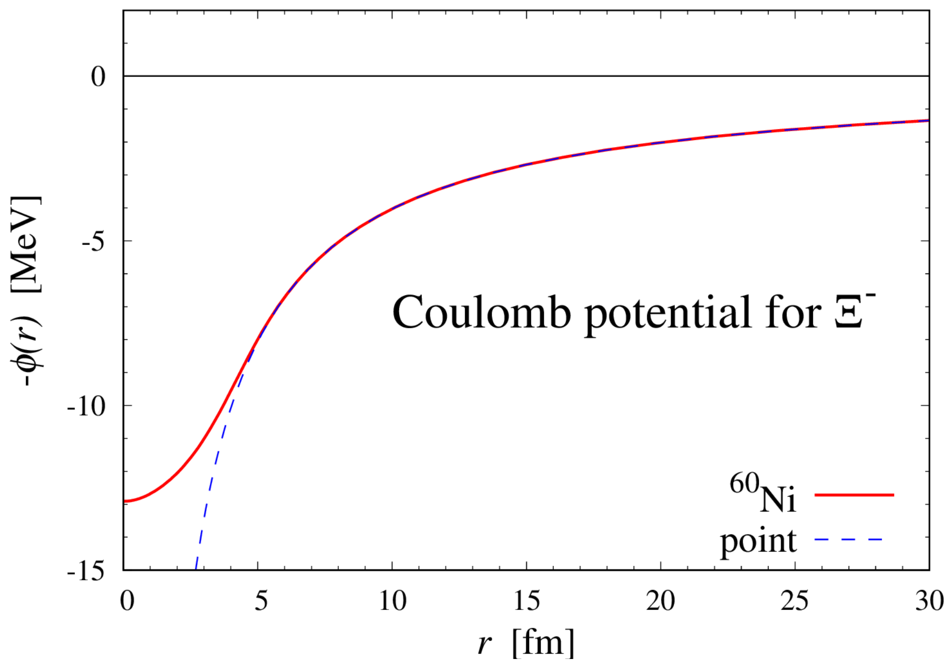
Ξ-atom w/o the strong interaction

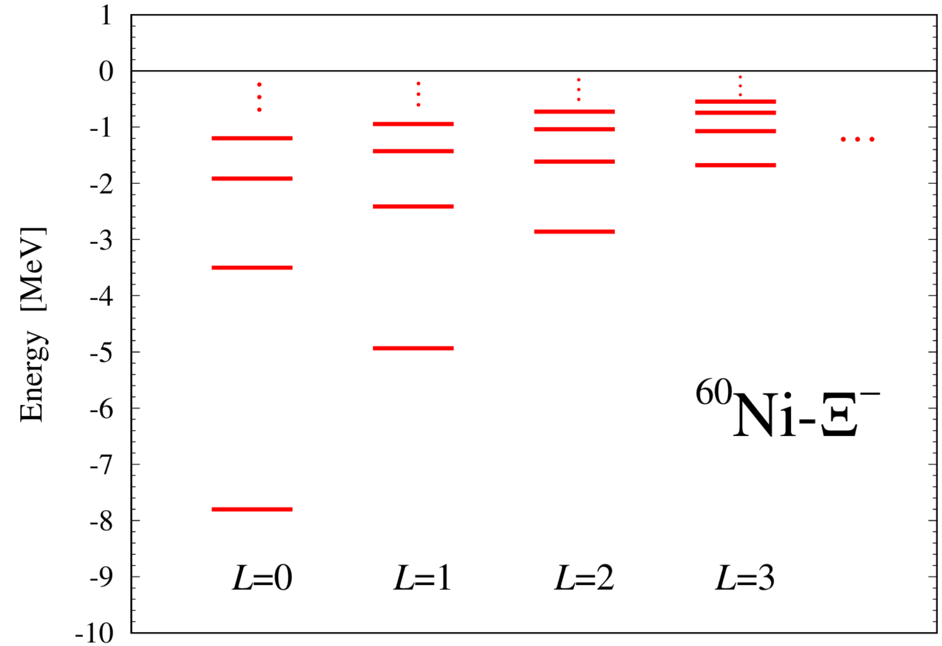




- by using a theoretical chage density of $^{60}\mathrm{Ni}$ for example
  - Skyrme HF w/ the parameter set SIII, i.e. no paring effect
  - $^{60}\mathrm{Ni}$ is chosen just so that proton number is around 30.
- We get many(infinit) coulomb bound states. = Ξ-atom

Ξ-atom w/o the strong interaction





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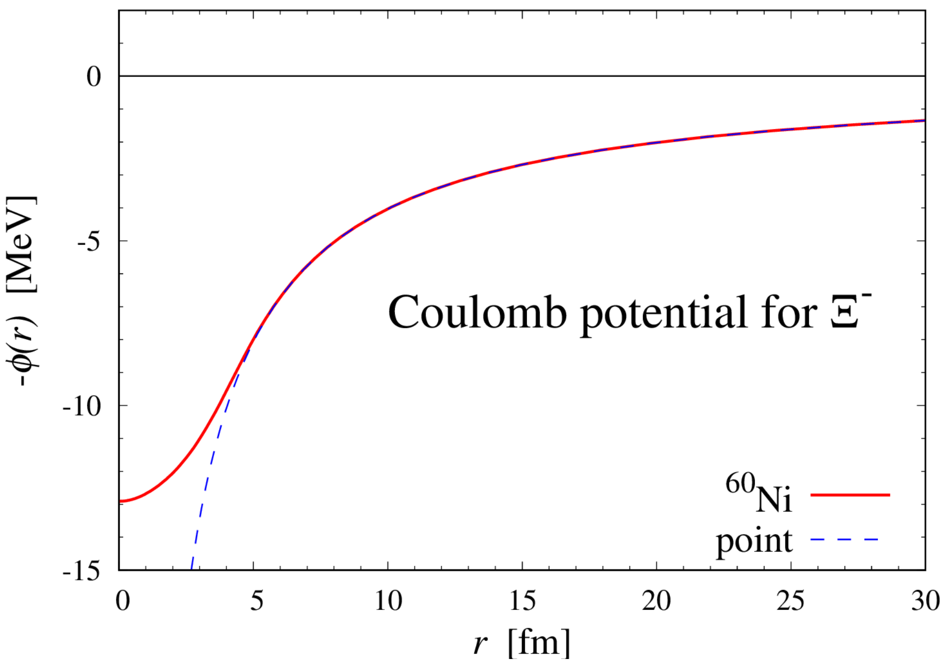
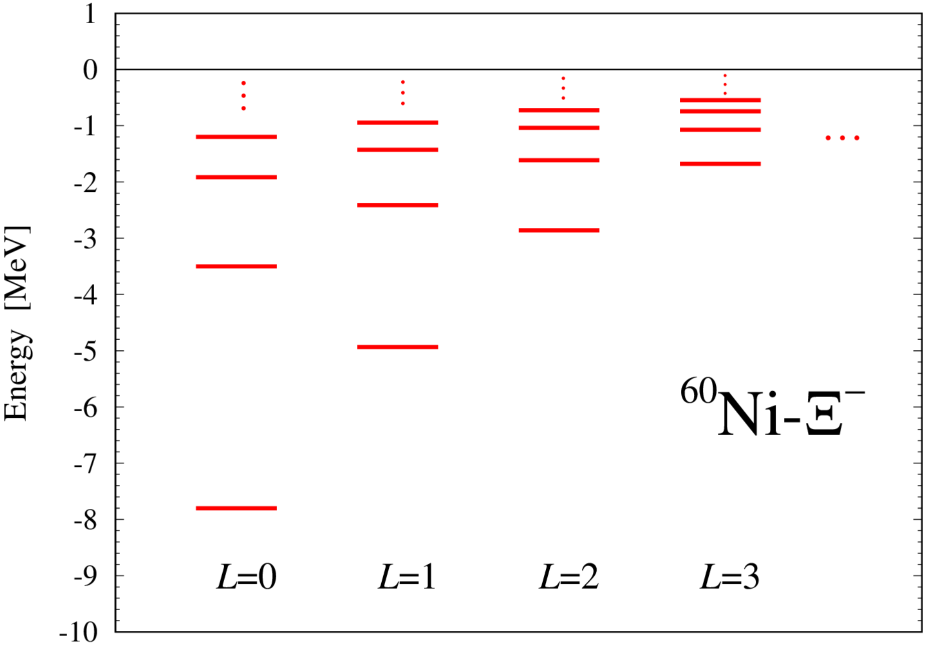
- Experimentalists can extract the shifts by measuring X-ray.

Ξ-atom w/o the strong interaction





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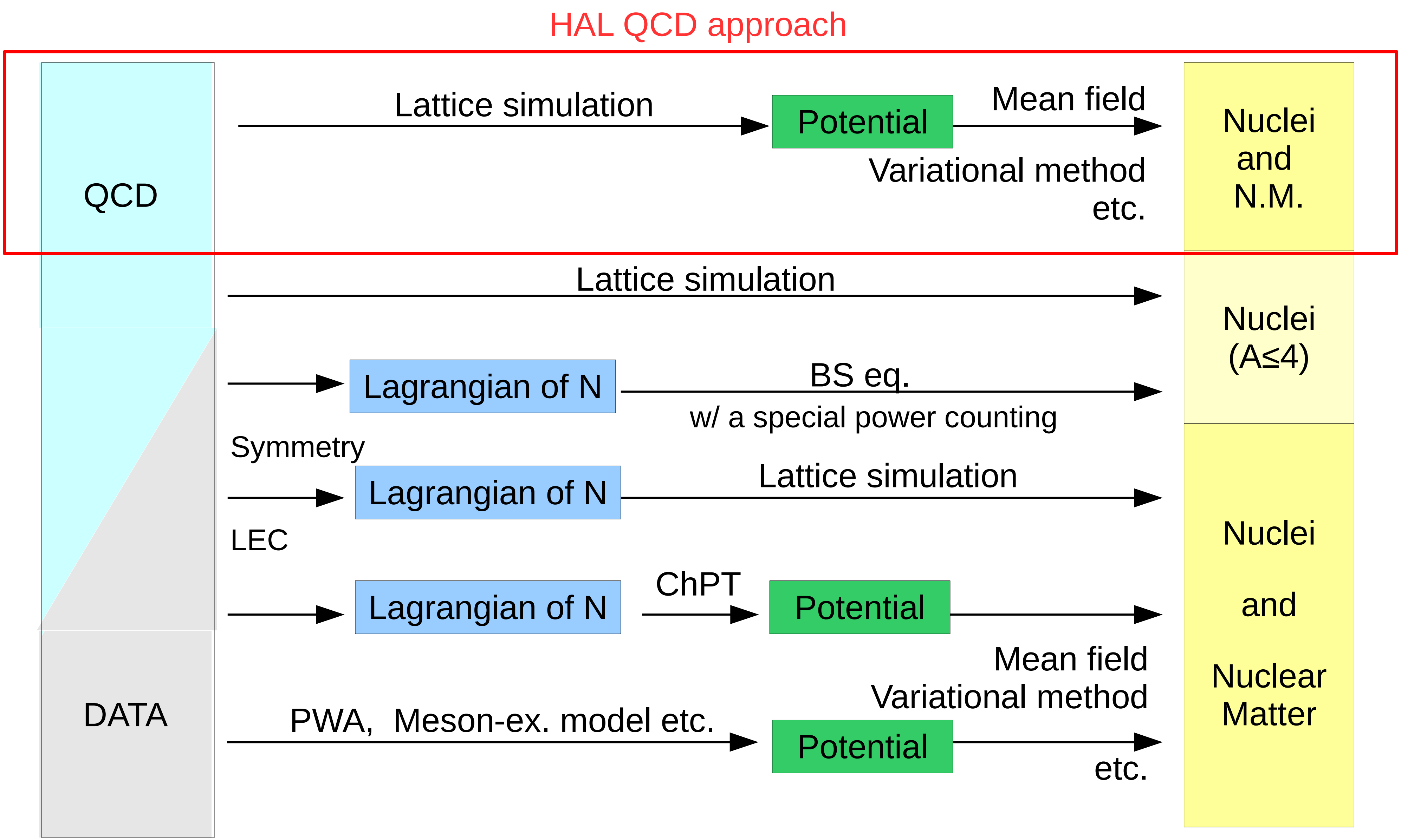
- We get many(infinit) coulomb bound states. = Ξ-atom
- In reality, these levels will be shifted by the strong interaction.
- Experimentalists can extract the shifts by measuring X-ray.
- So, we can study/check ΞN interaction through Ξ-atom.
  - atractive/repulsive? How strong/weak? = Goal of this study

Outline

1. Introduction
2. HAL QCD approach and method
   - S=−2 BB interactions from QCD
3. Application to strange nuclear physics
   - single-particle potential of Ξ in nuclear matter
   - Ξ-atom and Ξ-nucleus (Preliminary)
4. Summary and outlook

HAL QCD approach and method
$S=-2$ BB interactions from QCD

Various approaches in nuclear phys.



HAL method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89 (2010)
N. Ishii etal. [HAL QCD coll.] Phys. Lett. B712 , 437 (2012)

NBS wave function $\phi_{\vec{k}}(\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(\vec{x}+\vec{r},t) B_j(\vec{x},t) | B=2, \vec{k} \rangle$

DEFINE a potential U for all E eigenstates through a “Schrödinger eq.”

$$\left[-\frac{\nabla^2}{2\mu} \right] \phi_{\vec{k}}(\vec{r}) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \phi_{\vec{k}}(\vec{r}\,') = E_{\vec{k}} \phi_{\vec{k}}(\vec{r})$$

Non-local but
energy independent

4-point function $G(\vec{x},\vec{y},t-t_0) = \langle 0 | B_i(\vec{x},t) B_j(\vec{y},t)\ J(t_0) | 0 \rangle$

We measure $\psi(\vec{r},t) = \sum_{\vec{x}} G(\vec{x}+\vec{r},\vec{x},\ t-t_0) = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots$

$$\left[2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,') \psi(\vec{r}\,',t) = -\frac{\partial}{\partial t} \psi(\vec{r},t)$$

∇ expansion
& truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,') V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,') [V(\vec{r}) + \cancel{\nabla + \nabla^2} \ldots]$$

Therefor, in
the leading

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

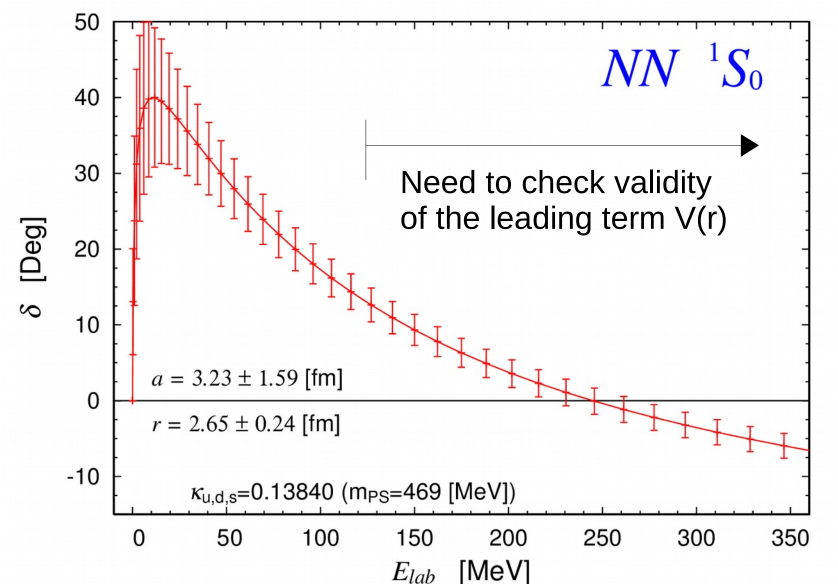
Multi-hadron in LQCD

- Direct : utilize temporal correlator and eigen-energy
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states
- HAL : utilize spatial correlation and "potential" $V(r)$ + ...

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: 4-point function contains NBS w.f.

- Advantages
  - No need to separate E eigenstate. Just need to measure $\psi(\vec{r},t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to $V=\infty$.
  - Can output more observables.



★ We can attack Ξ-atom too!!

LQCD simulation setup

- Nf = 2+1 full QCD
  - Clover fermion + Iwasaki gauge w/ stout smearing
  - Volume $96^4 \simeq (8\ \mathrm{fm})^4$ large enough to accommodate *BB* interaction
  - 1/a = 2333 MeV, a = 0.0845 fm

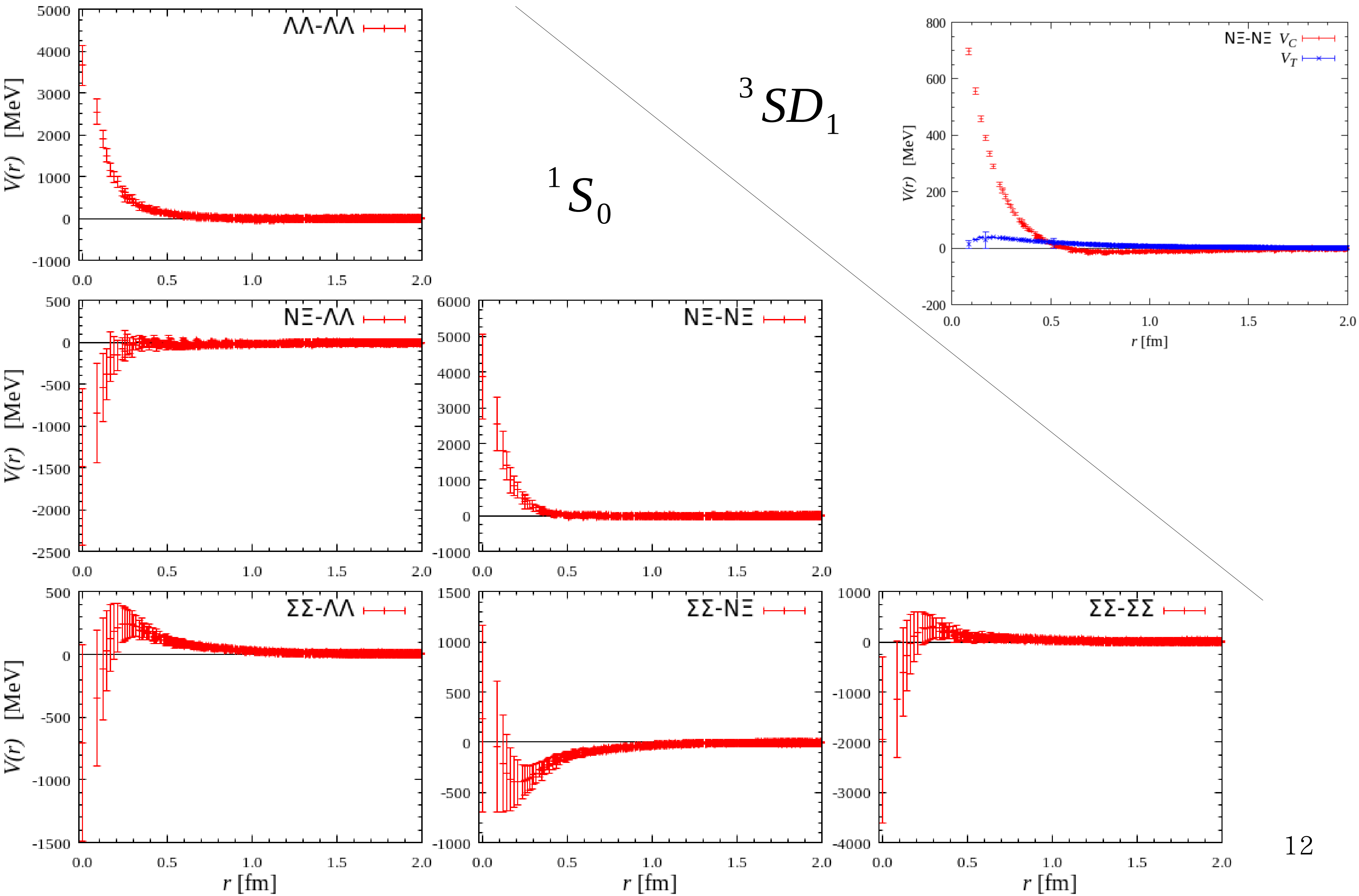
K-configuration

  - $M_\pi \simeq 146$, $M_K \simeq 525$ MeV almost physical point
    $M_N \simeq 956$, $M_\Lambda \simeq 1121$, $M_\Sigma \simeq 1201$, $M_\Xi \simeq 1328$ MeV
  - Collaboration in HPCI Strategic Program Field 5 Project 1

- Measurement
  - 4pt correlators: 52 channels in 2-octet-baryon (+ others)
  - Wall source w/ Coulomb gauge fixing
  - Dirichlet temporal BC to avoid the wrap around artifact
  - #data = 414 confs $\times$ 4 rot $\times$ (96,96) src.

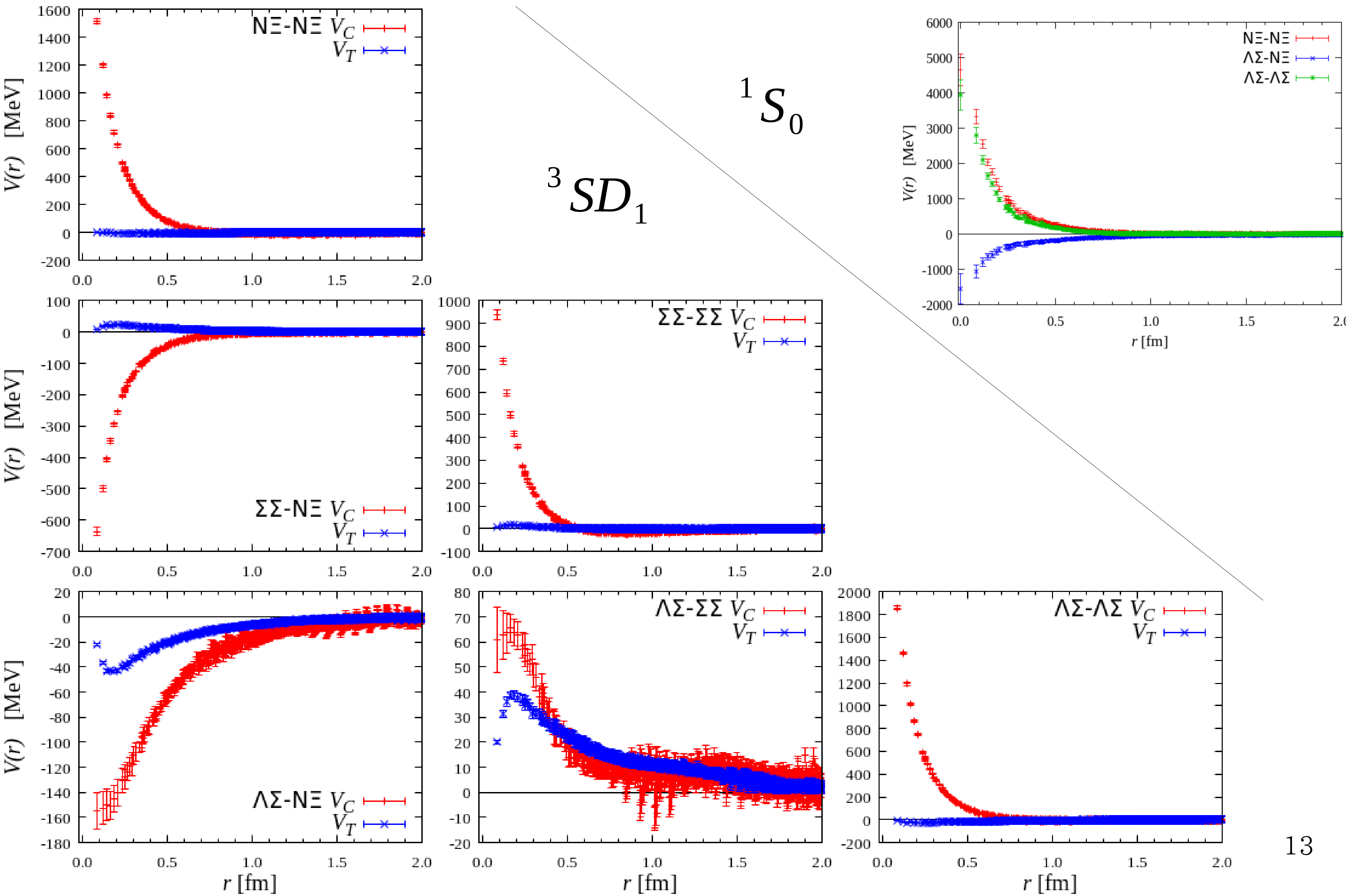
S=−2, I=0, BB potentials

(96,96) src
t−t0 = 12



S=−2, I=1, BB potentials

(96,96) src
$t - t_0$ = 12



single-particle potential Ξ
in nuclear matter

single-particle potential Ξ in nuclear matter

spectrum: $e_{\Xi}(k;\rho) = \dfrac{k^2}{2M_{\Xi}} + \underline{U_{\Xi}(k;\rho)}$

This U is important quantity determining chemical potential of particle in matter

Brueckner-Hartree-Fock

- single-particle potential of Ξ

$$U_{\Xi}(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \le k_F} \langle k k' | G^{SLJ}_{(\Xi N)(\Xi N)}\left(e_{\Xi}(k) + e_N(k')\right) | k k' \rangle$$

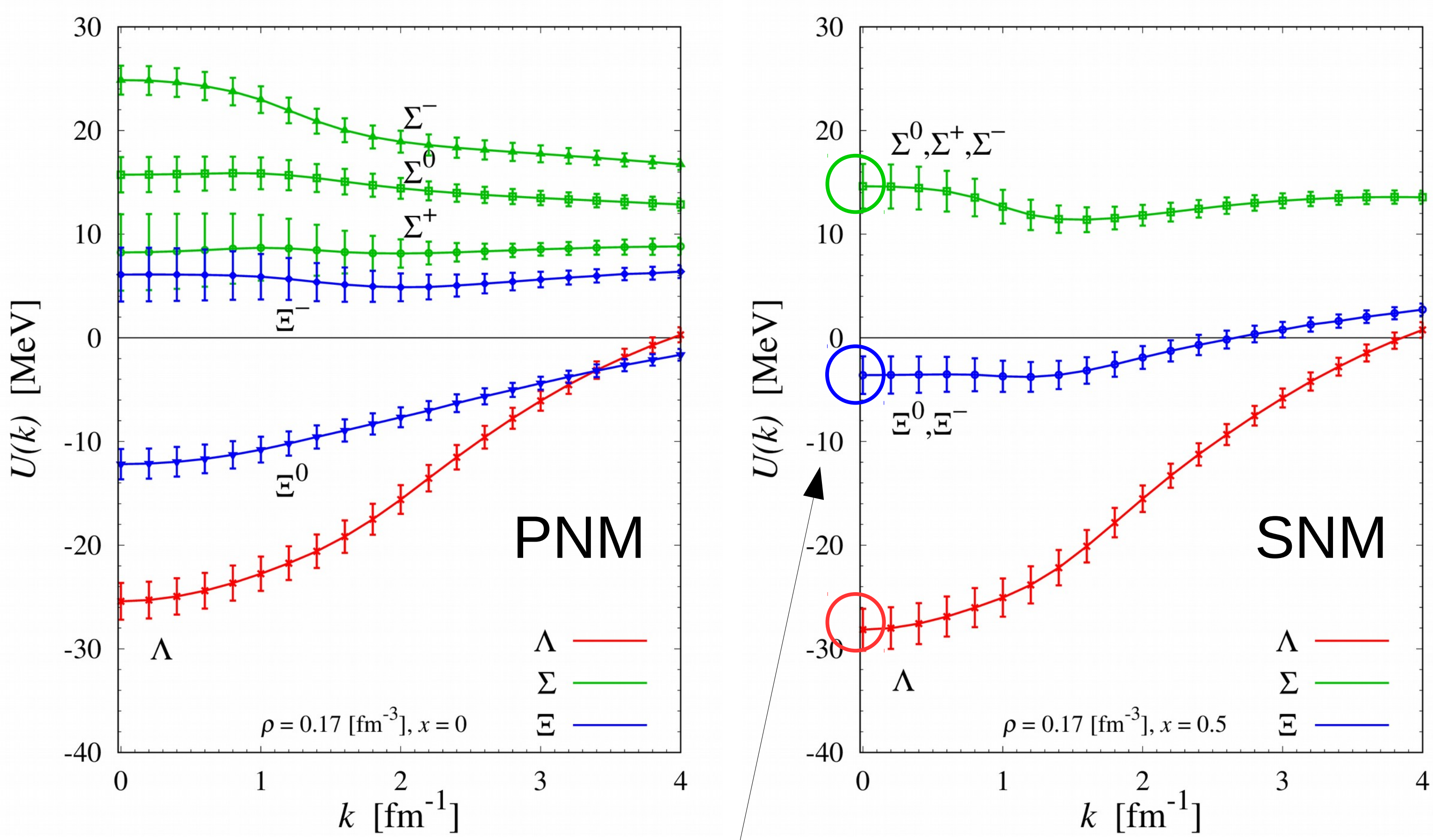
- ΞN G-matrix using $M^{\mathrm{Phys}}_{N,Y}$, $U^{\mathrm{AV18+UIX}}_{n,p}$, $U^{\mathrm{LQCD}}_{\Lambda,\Sigma}$, $V^{\mathrm{LQCD}}_{S=-2}$, U^{LQCD}_{Ξ}

Flavor symmetric $^1S_0$ sectors

$$Q=0 \quad \begin{pmatrix} G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+\Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0\Sigma^0)} & G_{(\Xi^0 n)(\Sigma^0\Lambda)} & G_{(\Xi^0 n)(\Lambda\Lambda)} \\ G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+\Sigma^-)} & G_{(\Xi^- p)(\Sigma^0\Sigma^0)} & G_{(\Xi^- p)(\Sigma^0\Lambda)} & G_{(\Xi^- p)(\Lambda\Lambda)} \\ G_{(\Sigma^+\Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+\Sigma^-)(\Xi^- p)} & G_{(\Sigma^+\Sigma^-)(\Sigma^+\Sigma^-)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Sigma^0)} & G_{(\Sigma^+\Sigma^-)(\Sigma^0\Lambda)} & G_{(\Sigma^+\Sigma^-)(\Lambda\Lambda)} \\ G_{(\Sigma^0\Sigma^0)(\Xi^0 n)} & G_{(\Sigma^0\Sigma^0)(\Xi^- p)} & G_{(\Sigma^0\Sigma^0)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Sigma^0)(\Sigma^0\Sigma^0)} & G_{(\Sigma^0\Sigma^0)(\Sigma^0\Lambda)} & G_{(\Sigma^0\Sigma^0)(\Lambda\Lambda)} \\ G_{(\Sigma^0\Lambda)(\Xi^0 n)} & G_{(\Sigma^0\Lambda)(\Xi^- p)} & G_{(\Sigma^0\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Sigma^0)} & G_{(\Sigma^0\Lambda)(\Sigma^0\Lambda)} & G_{(\Sigma^0\Lambda)(\Lambda\Lambda)} \\ G_{(\Lambda\Lambda)(\Xi^0 n)} & G_{(\Lambda\Lambda)(\Xi^- p)} & G_{(\Lambda\Lambda)(\Sigma^+\Sigma^-)} & G_{(\Lambda\Lambda)(\Sigma^0\Sigma^0)} & G_{(\Lambda\Lambda)(\Sigma^0\Lambda)} & G_{(\Lambda\Lambda)(\Lambda\Lambda)} \end{pmatrix}$$

$$Q=+1 \quad \begin{pmatrix} G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+\Lambda)} \\ G_{(\Sigma^+\Lambda)(\Xi^0 p)} & G_{(\Sigma^+\Lambda)(\Sigma^+\Lambda)} \end{pmatrix} \qquad Q=-1 \quad \begin{pmatrix} G^{SLJ}_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^-\Lambda)} \\ G_{(\Sigma^-\Lambda)(\Xi^- n)} & G_{(\Sigma^-\Lambda)(\Sigma^-\Lambda)} \end{pmatrix}$$

Hyperon single-particle potentials



$@\rho = 0.17\,[\mathrm{fm}^{-3}]$

U_Λ and U_Σ are obtained with LQCD $V^{S=-1}$ assuming the SU(3)$_F$ symm.

- QCD leads that Ξ feels attraction ~4 MeV in SNM
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10\,?, \quad U_\Sigma^{\mathrm{Exp}}(0) \geq +20\,? \quad [\mathrm{MeV}]$$

attraction　　attraction small　　repulsion

Hyperon single-particle potentials

- Breakdown of $U_Y(0\,;\rho_0)$ in SNM including spin,iso-spin multiplicity

	I=1/2						total
Λ	$^1S_0$	$^3S_1$	$^3D_1$				
	−3.49	−24.84	0.18				−28.16

	I=1/2			I=3/2			total
Σ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	7.43	−9.28	0.07	−4.97	21.80	−0.43	14.62

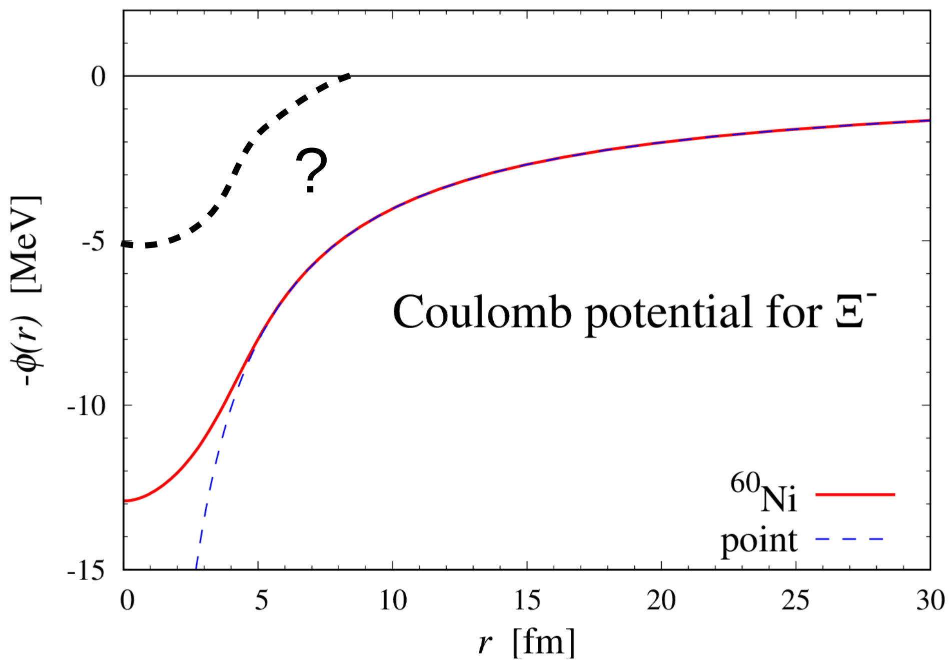
	I=0			I=1			total
Ξ	$^1S_0$	$^3S_1$	$^3D_1$	$^1S_0$	$^3S_1$	$^3D_1$	
	−4.48	−4.37	−0.01	9.08	−3.74	−0.08	−3.60

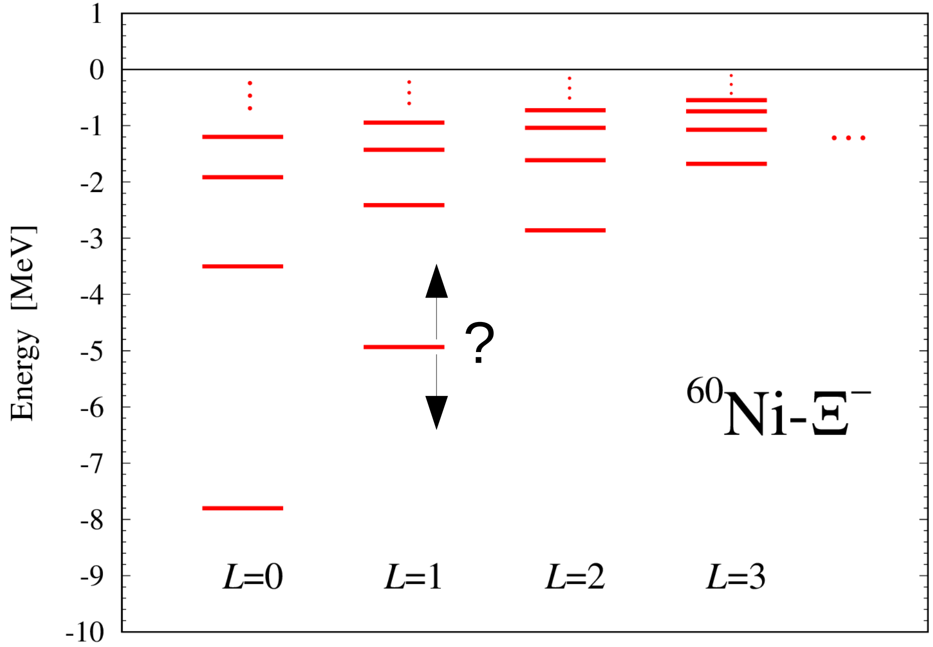
All ΞN S-wave interactions provide attraction in SNM
except for I=1, S=0 chanel

Our prediction based on QCD

Ξ-atom and Ξ-nucleus

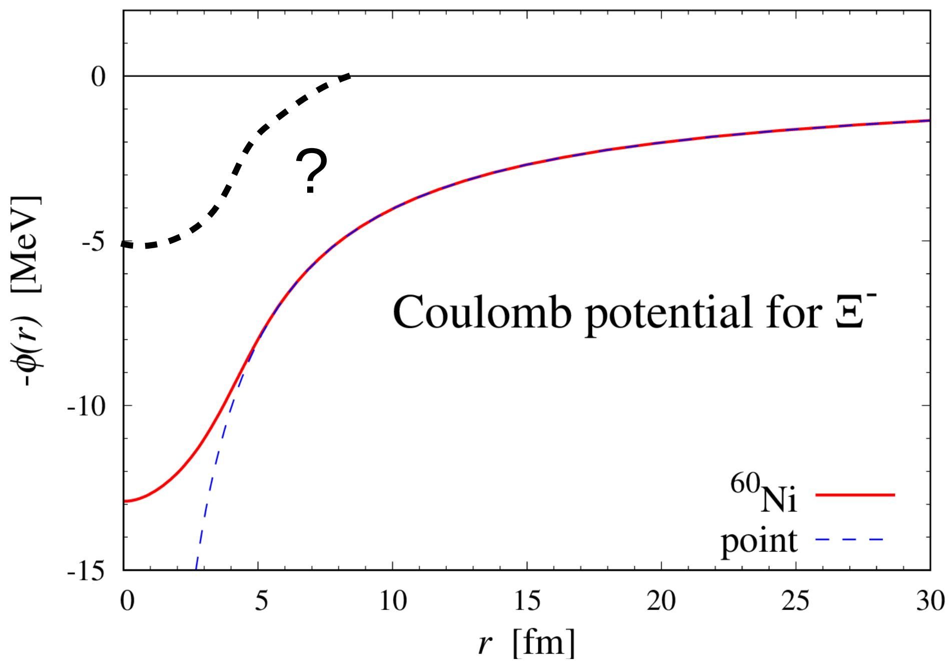
ΞN *G*-matrix potential

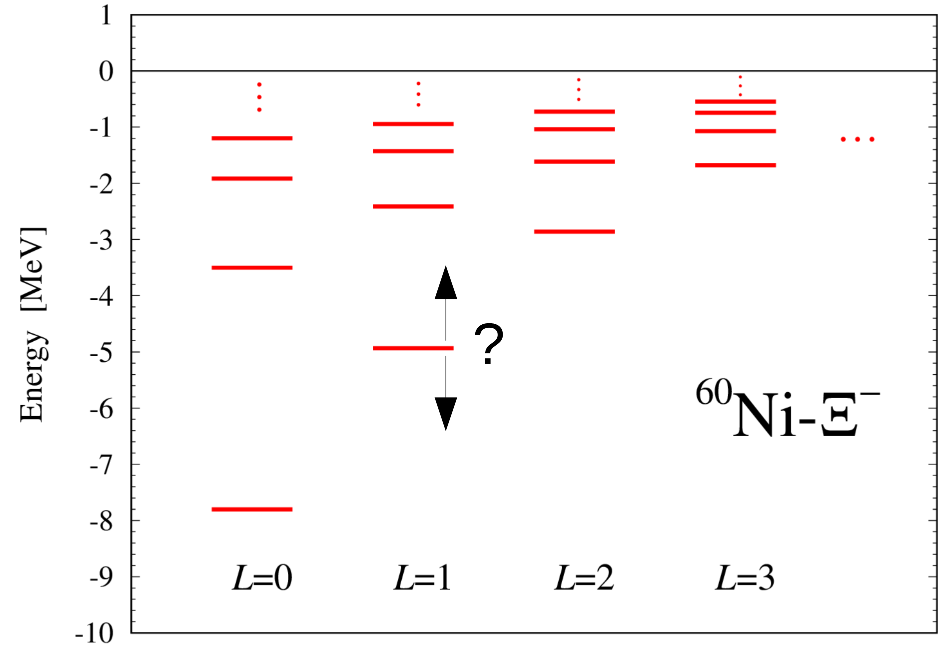




- We want to add ^{60}Ni-Ξ strong potential to that calc.

ΞN G-matrix potential





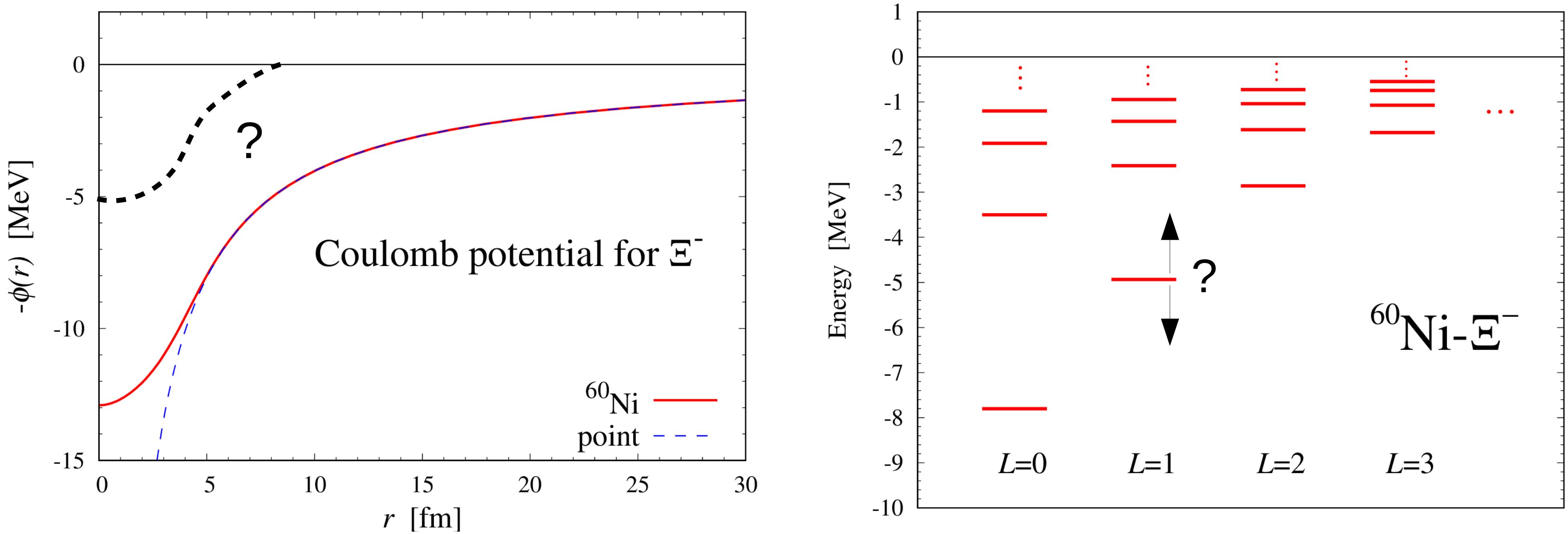
- We want to add ^{60}Ni-Ξ strong potential to that calc.
- We employ the "G-matrix potential" which is a local potential made so that simulate on-shell $G^{SLJ}_{\Xi N}(k,k)$

$$\frac{2}{\pi}\int_0^\infty r^2 j_L(kr)\,\widetilde{G}^{SLJ}(r)\,j_L(kr) \approx G^{SLJ}(k,k)$$

$$\widetilde{G}^{SLJ}(r) = C_1 e^{-b_1 r^2} + C_2 e^{-b_2 r^2} + C_3 e^{-b_3 r^2}$$

- Y. Yamamto, T. Motoba, and T. Rijken, Prog. Theo. Supp. No.185, 2010

ΞN G-matrix potential



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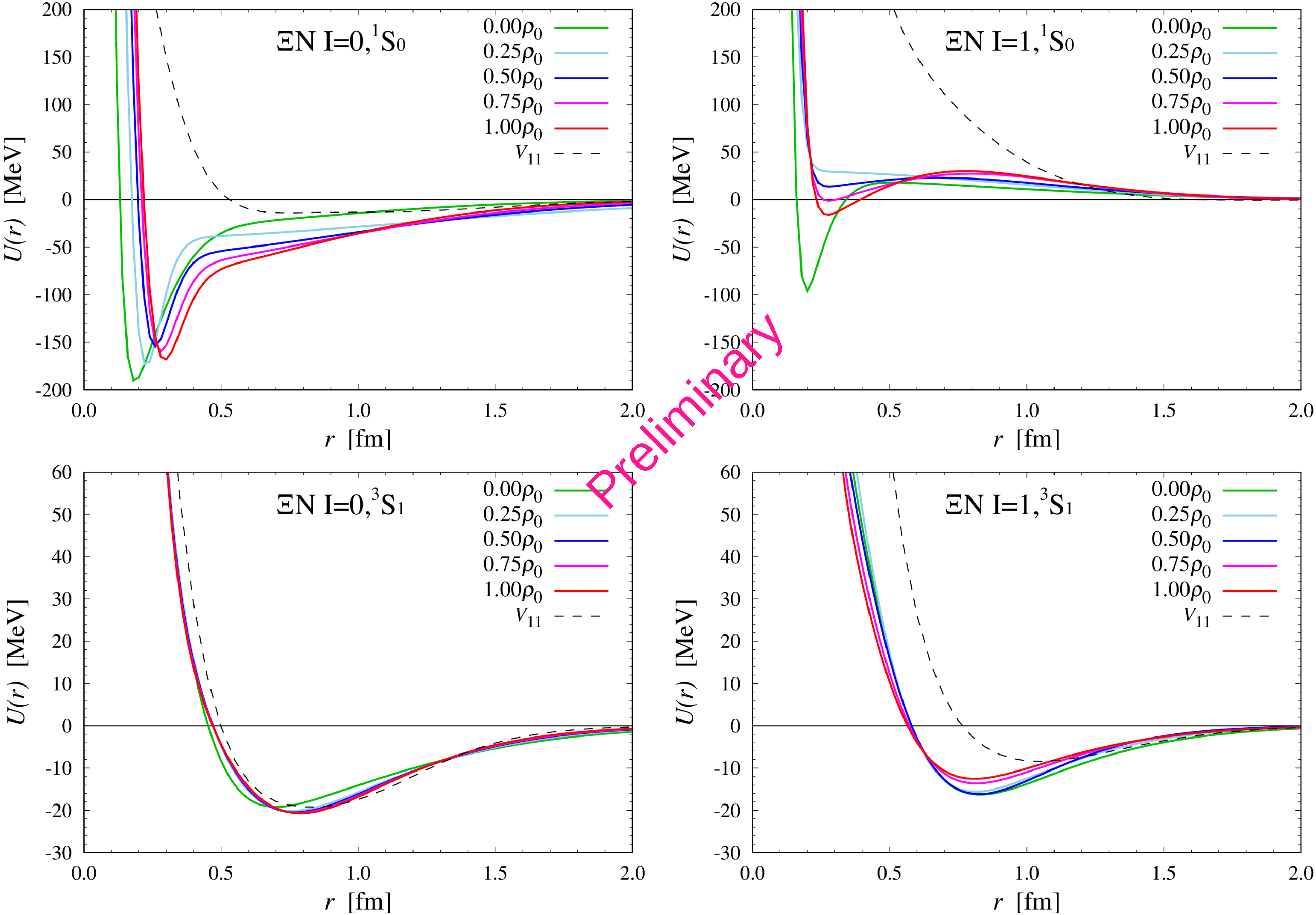
$$\tilde{G}^{SLJ}(r) = C_1 e^{-b_1 r^2} + C_2 e^{-b_2 r^2} + C_3 e^{-b_3 r^2}$$

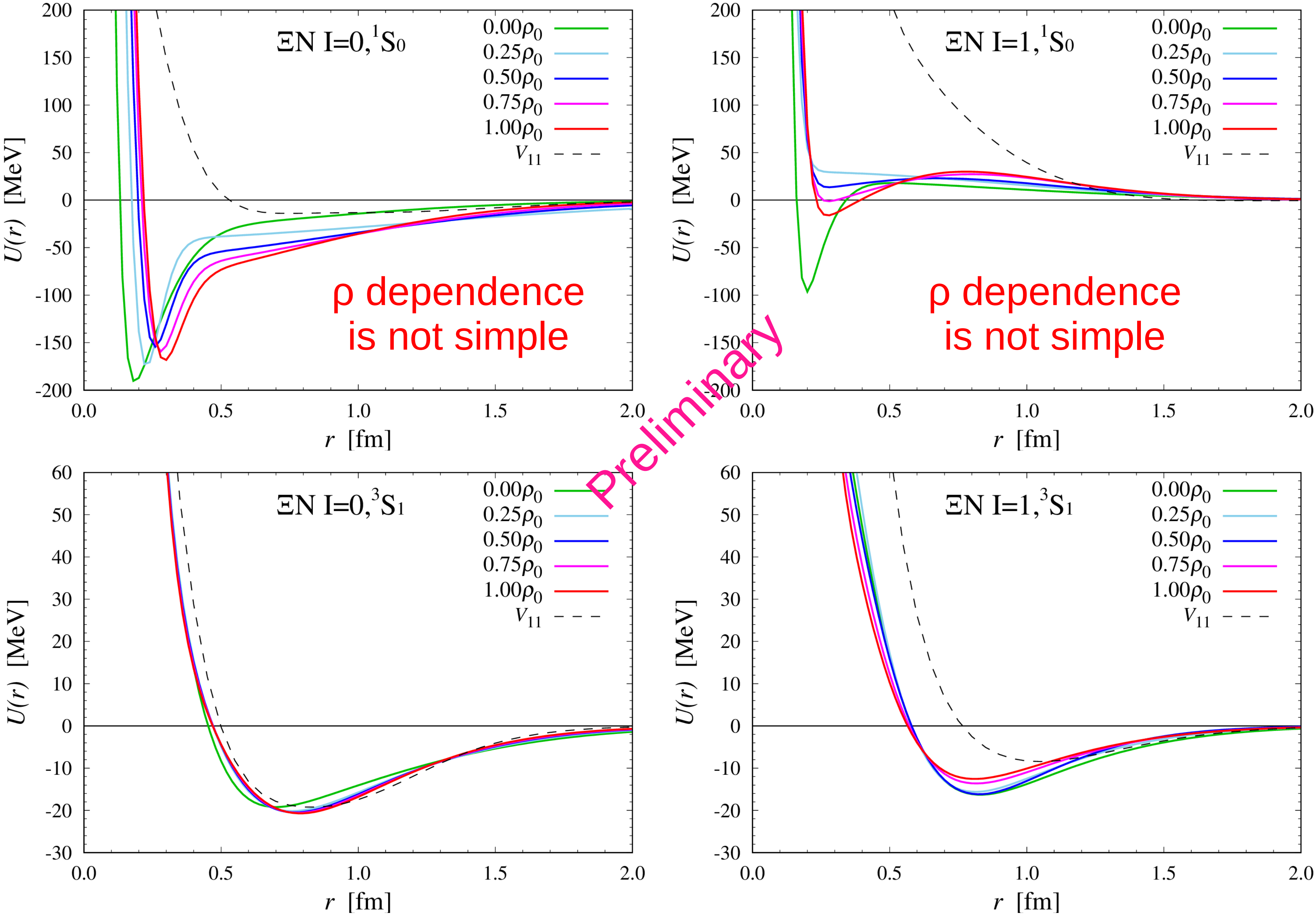
We neglect the imaginary part of G this time

- Y. Yamamto, T. Motoba, and T. Rijken, Prog. Theo. Supp. No.185, 2010

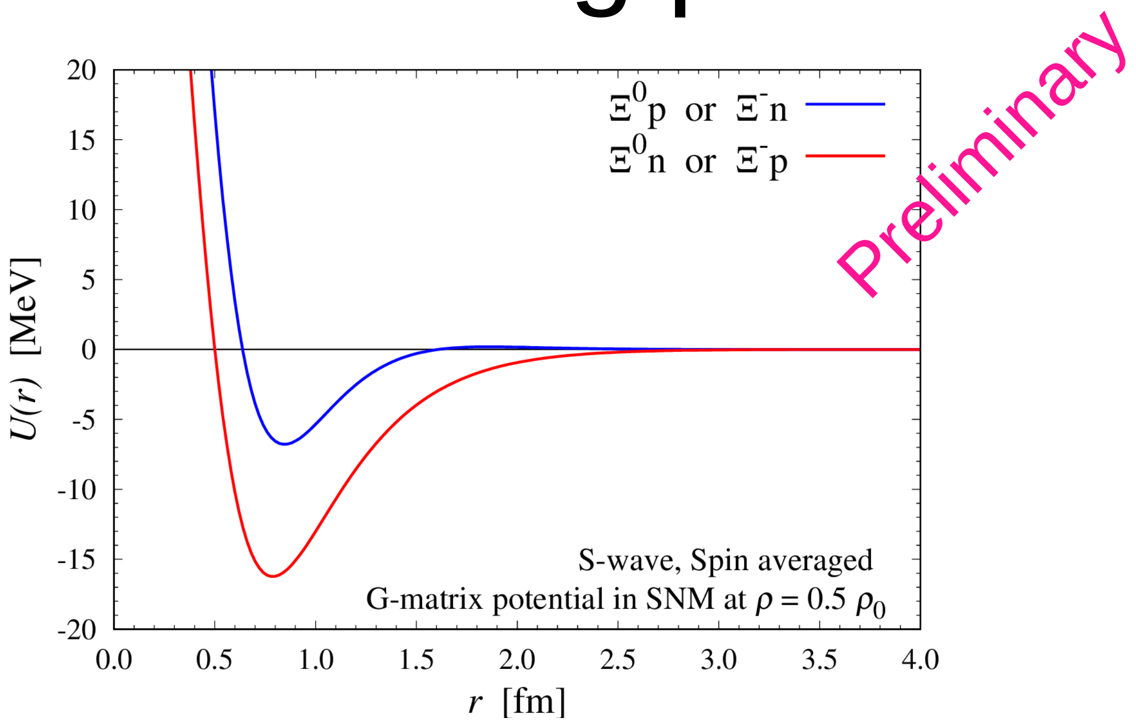
ΞN G-matrix potential in SNM



ΞN *G*-matrix potential in SNM

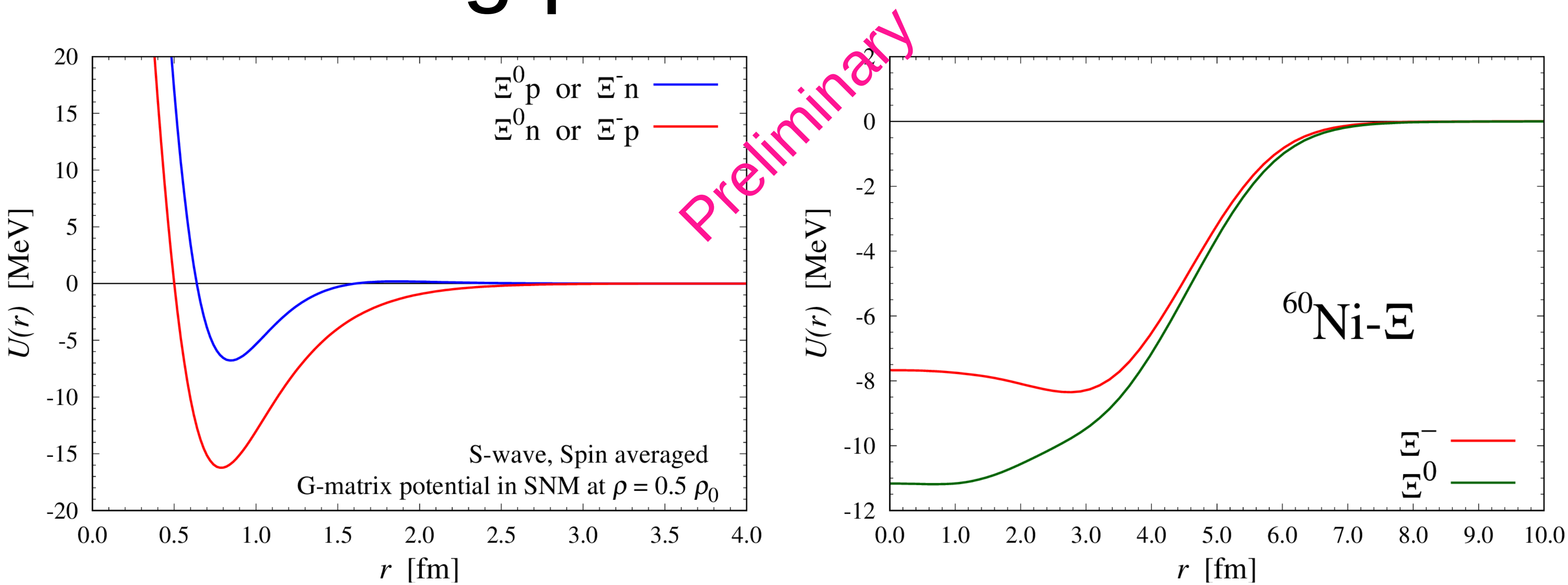


^{60}Ni-Ξ strong potential



- Chage base, spin averaged ΞN G-matrix potential at $0.5\rho_0$

^{60}Ni-Ξ strong potential

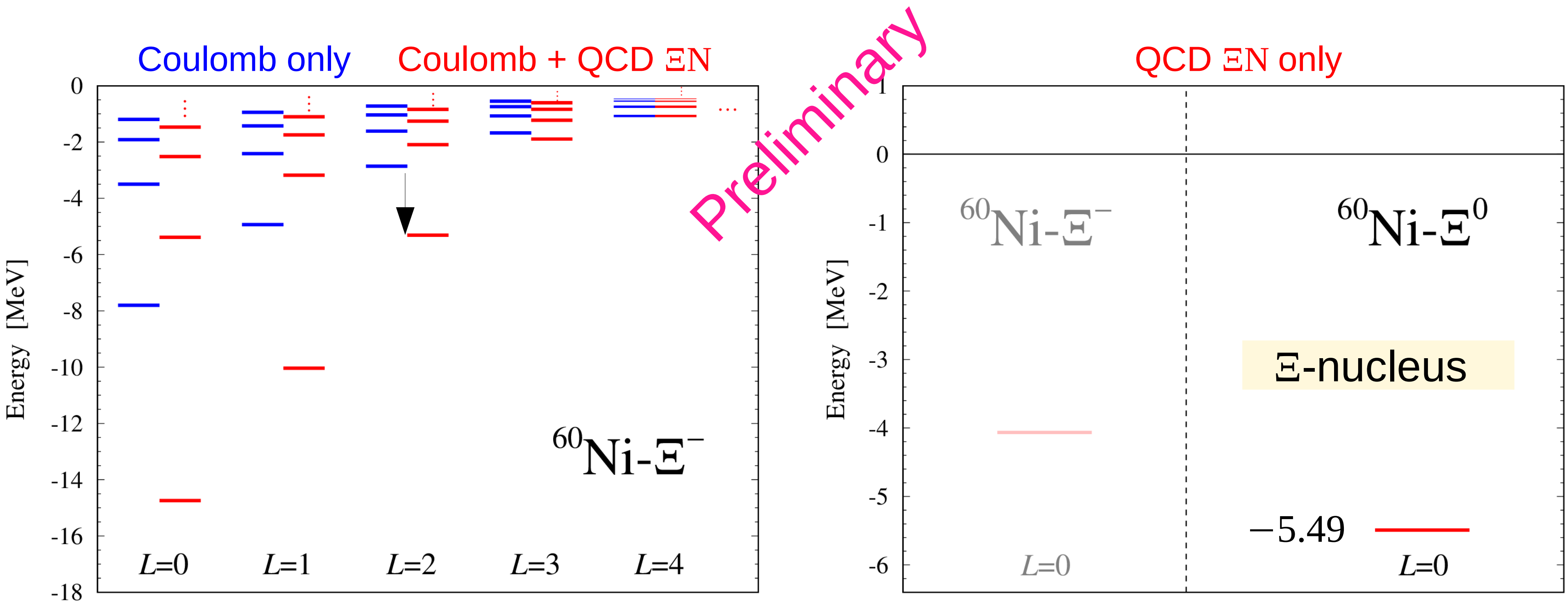


- Chage base, spin averaged ΞN G-matrix potential at 0.5ρ_0
- Single folding potential with ^{60}Ni proton/neurton density

$$U(\vec{r})_{\text{Ni-}\Xi} = \int d^3\vec{r}\,'\ \rho^{p}_{\text{Ni}}(\vec{r}\,')\ U_{\Xi p}(|\vec{r}-\vec{r}\,'|\,;\,\tilde{\rho}) + \int d^3\vec{r}\,'\ \rho^{n}_{\text{Ni}}(\vec{r}\,')\ U_{\Xi n}(|\vec{r}-\vec{r}\,'|\,;\tilde{\rho})$$

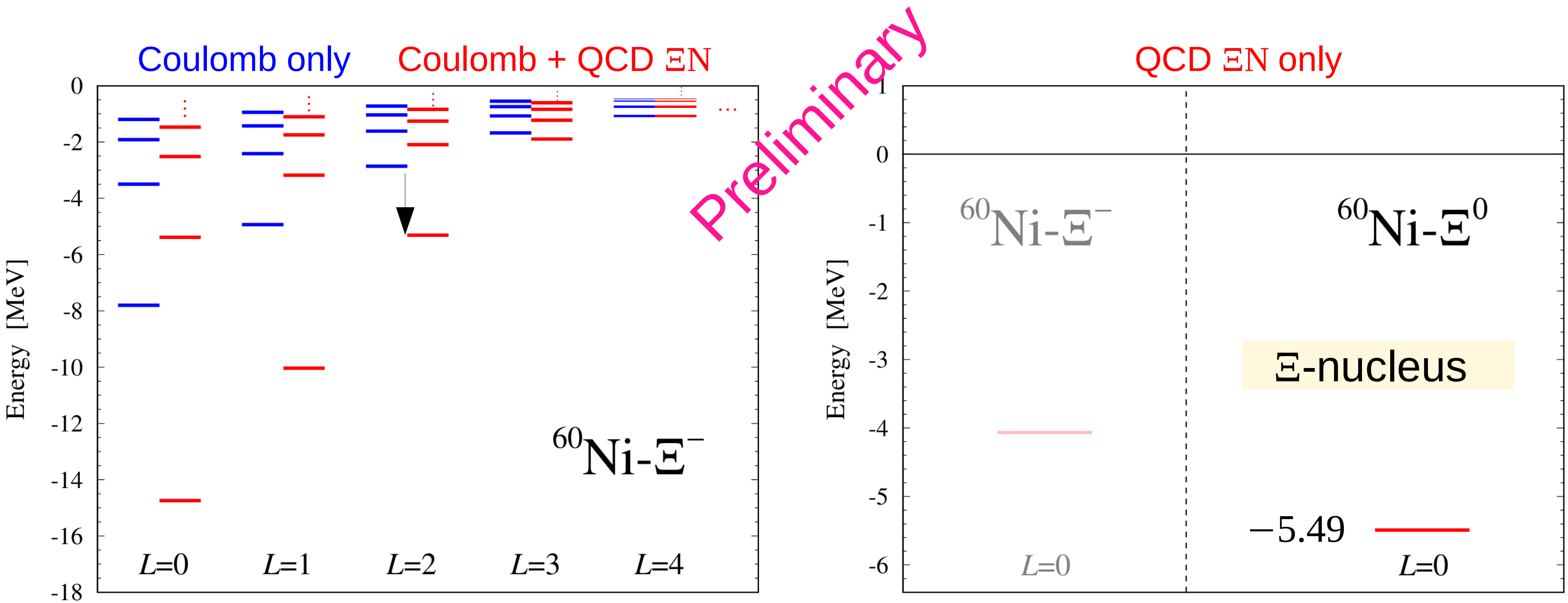
  - Fixed $\tilde{\rho} = 0.5\ \rho_0$ is used, for the moment, because of the complicaed ρ dependence of $U(r\,;\rho)$, ie. very primitive.

^{60}Ni-Ξ bound states



- We see downward energy shifts due to the QCD ΞN

^{60}Ni-Ξ bound states



- We see downward energy shifts due to the QCD ΞN
- But,...
  - S-wave spin-avaraged $U_{\Xi N}(r)$ may not be enough.
  - Imaginary potential $W_{\Xi N}(r)$ would be important.
  - Only shifts in larger L states are accessible in experiments…
    - eg. X-ray of transition from L=7 bttom to L=6 bottom
  - We need to improve our calculation more.

Summary and outlook

- We have all BB S-wave interaction from QCD
  - Lattice QCD simulation at a almost physical point
  - HAL QCD method ie. potentential method
- Especially, we have a theoretical ΞN inteaction
- Our QCD ΞN inteaction leads
  - attraction of Ξ in SNM at ρ_0 about $U_\Xi(0) = -4$ MeV
  - which seems qualitatively consistent with experiments
  - All S-wave ΞN interaction provide attraction except for $I=1, S=0$
- We' studied Ξ-atom to check our QCD ΞN
  - G-matrix potential + Folding = ^{60}Ni-Ξ strong potential
  - We obtained downward energy sifts as expected
  - In addtion, we found a nuclear bound state with 5 MeV binging
- But, we have to improve our calculation
  - Include imaginary part of G-matrix potential
  - Folding including density dependence of G-matrix pot.
  - Try very tiny shifts of larger L states, L=6,7….

Thank you !!

Backup

FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential *U(r,r')* or *V(r)* depends on energy?

FAQ

1. Does your potential depend on the choice of source?

→ No. Some sources may enhance excited states in 4-point func. However, it is no longer a problem in our new method.

2. Does your potential depend on choice of operator at sink?

→ Yes. It can be regarded as the "scheme" to define a potential. Note that a potential itself is not physical observable. We will obtain unique result for physical observables irrespective to the choice, as long as the potential $U(r,r')$ is deduced exactly.

FAQ

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

➔ By definition, $U(r,r')$ is non-local but energy independent. While, determination and validity of its leading term $V(r)$ depend on energy because of the truncation.

However, we know that the dependence in *NN* case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.} = 0 - 90$ MeV. We rely on this in our study.

If we find some dependence, we will determine the next leading term of the expansion from the dependence.

FAQ

in $SU(3)_F$ limit, ie. heavy u,d quark world

4. Is the H a compact six-quark object or a tight BB bound state?

FAQ

found in $SU(3)_F$ limit, ie. heavy u,d quark world

4. Is the H a compact six-quark object or a tight BB bound state?

➔ Both.
There is no distinct difference between two in QCD.
Note that baryon is made of three quarks in QCD.
Imagine a compact 6-quark object in $(0S)^6$ configuration.
This configuration can be re-written in a form of
$(0S)^3 \times (0S)^3 \times \mathrm{Exp}(-a\, r^2)$ with relative coordinate r.
This demonstrate that a compact six-quark object,
at the same time, has a BB type configuration.
In LQCD simulation at $SU(3)_F$ limits, we've established
existence of a $B=2, S=-2, I=0$ stable QCD eigenstate.