Study of Ξ-nucleus and Ξ-atom based on the ΞN interaction from QCD on lattice

Takashi Inoue  @Nihon University

for
HAL QCD Collaboration

S. Aoki  YITP Kyoto Univ.
Y. Akahoshi  YITP Kyoto Univ.
K. Sasaki  YITP Kyoto Univ.
T. Doi  RIKEN Nishina
T. M. Doi  RIKEN Nishina
S. Gongyo  RIKEN Nishina
T. Hatsuda  RIKEN Nishina
T. Sugiura  RIKEN Nishina
Y. Ikeda  RCNP Osaka Univ
N. Ishii  RCNP Osaka Univ.
K. Murano  RCNP Osaka Univ.
H. Nemura  RCNP Osaka Univ.
T. Aoyama  KEK Theory Center
F. Etminan  Univ. Birjand
T. I.  Nihon Univ.

INPC, July 29 2019, Glasgow UK
Introduction
$\Xi$-atom \textit{w/o} the strong interaction

- by using a theoretical charge density of $^{60}\text{Ni}$ for example
  - Skyrme HF \textit{w/} the parameter set SIII, i.e. \textit{no} paring effect
  - $^{60}\text{Ni}$ is chosen just so that proton number is around 30.
- We get many (infinit) coulomb \textit{bound states}. $= \Xi$-atom
by using a theoretical change density of $^{60}\text{Ni}$ for example
  - Skyrme HF w/ the parameter set SIII, i.e. no pairing effect
  - $^{60}\text{Ni}$ is chosen just so that proton number is around 30.
- We get many (infinit) coulomb bound states. = Ξ-atom
- In reality, these levels will be shifted by the strong interaction.
- Experimentalists can extract the shifts by measuring X-ray.
\(|\Xi\)-atom w/o the strong interaction

- by using a theoretical charge density of \(^{60}\text{Ni}\) for example
  - Skyrme HF w/ the parameter set SIII, i.e. no pairing effect
  - \(^{60}\text{Ni}\) is chosen just so that proton number is around 30.
- We get many (infinite) coulomb bound states. \(= \Xi\)-atom
- In reality, these levels will be shifted by the strong interaction.
- Experimentalists can extract the shifts by measuring X-ray.
- So, we can study/check \(\Xi N\) interaction through \(\Xi\)-atom.
  - attractive/repulsive? How strong/weak? \(= \text{Goal of this study}\)
Outline

1. Introduction

2. HAL QCD approach and method
   • S=−2 BB interactions from QCD

3. Application to strange nuclear physics
   • single-particle potential of $\Xi$ in nuclear matter
   • $\Xi$-atom and $\Xi$-nucleus (Preliminary)

4. Summary and outlook
HAL QCD approach and method
$S=-2$ BB interactions from QCD
Various approaches in nuclear phys.

HAL QCD approach

QCD

Lattice simulation → Potential → Mean field

Variational method etc.

Nuclei and N.M.

Nuclei (A ≤ 4)

Symmetry

LEC

Lagrangian of N

BS eq. w/ a special power counting

Lattice simulation

Lagrangian of N

Lattice simulation

Lagrangian of N

ChPT

Potential

Mean field Variational method etc.

Potential

DATA

PWA, Meson-ex. model etc.
HAL method

NBS wave function \( \phi_{\vec{k}}(\vec{r}) = \sum_{\vec{\chi}} \langle 0 | B_i(\vec{\chi} + \vec{r}, t)B_j(\vec{\chi}, t) | B=2, \vec{k} \rangle \)

**DEFINE** a potential \( U \) for all \( E \) eigenstates through a “Schrödinger eq.”

\[
\left[ -\frac{\nabla^2}{2\mu} \right] \phi_{\vec{k}}(\vec{r}) + \int d^3 \vec{r}' U(\vec{r}', \vec{r}') \phi_{\vec{k}}(\vec{r}') = E_{\vec{k}} \phi_{\vec{k}}(\vec{r})
\]

4-point function \( G(\vec{x}, \vec{y}, t - t_0) = \langle 0 | B_i(\vec{x}, t)B_j(\vec{y}, t) J(t_0) | 0 \rangle \)

We measure \( \psi(\vec{r}, t) = \sum_{\vec{\chi}} G(\vec{\chi} + \vec{r}, \vec{\chi}, t - t_0) = \sum_{\vec{k}} A_{\vec{k}} \phi_{\vec{k}}(\vec{r}) e^{-W_{\vec{k}}(t-t_0)} + \cdots \)

\[
\left[ 2M_B - \frac{\nabla^2}{2\mu} \right] \psi(\vec{r}, t) + \int d^3 \vec{r}' U(\vec{r}, \vec{r}') \psi(\vec{r}', t) = -\frac{\partial}{\partial t} \psi(\vec{r}, t)
\]

\( \nabla \) expansion & truncation

\[
U(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') V(\vec{r}, \nabla) = \delta(\vec{r} - \vec{r}') [V(\vec{r}) + \nabla + \nabla^2 + \cdots]
\]

Thereof, in the leading

\[
V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\partial}{\partial t} \frac{\psi(\vec{r}, t)}{\psi(\vec{r}, t)} - 2M_B
\]

Multi-hadron in LQCD

- Direct: utilize **temporal** correlator and **eigen-energy**
  - Lüscher's finite volume method for phase-shifts
  - Infinite volume extrapolation for bound states

- HAL: utilize **spatial** correlation and “potential” $V(r) + \ldots$

$$V(\vec{r}) = \frac{1}{2\mu} \frac{\nabla^2 \psi(\vec{r}, t)}{\psi(\vec{r}, t)} - \frac{\partial}{\partial t} \frac{\psi(\vec{r}, t)}{\psi(\vec{r}, t)} - 2 M_B \psi(\vec{r}, t): 4\text{-point function contains NBS w.f.}$$

- Advantages
  - **No need to separate** $E$ eigenstate. Just need to measure $\psi(\vec{r}, t)$
  - Then, potential can be extracted.
  - Demand a minimal lattice volume. No need to extrapolate to $V=\infty$.
  - Can output more observables.

☆ We can attack $\Xi$-atom too!!
LQCD simulation setup

- Nf = 2+1 full QCD
  - Clover fermion + Iwasaki gauge w/ stout smearing
  - Volume $96^4 \approx (8 \text{ fm})^4$ large enough to accommodate $BB$ interaction
  - $1/a = 2333 \text{ MeV, } a = 0.0845 \text{ fm}$ K-configuration
  - $M_\pi \approx 146, M_K \approx 525 \text{ MeV}$ almost physical point
    - $M_N \approx 956, M_\Lambda \approx 1121, M_\Sigma \approx 1201, M_\Xi \approx 1328 \text{ MeV}$
  - Collaboration in HPCI Strategic Program Field 5 Project 1

- Measurement
  - 4pt correlators: 52 channels in 2-octet-baryon (+ others)
  - Wall source w/ Coulomb gauge fixing
  - Dirichlet temporal BC to avoid the wrap around artifact
  - #data = 414 confs $\times$ 4 rot $\times$ (96,96) src.
$S=−2, \ I=0, \ BB \ potentials$

$\Lambda\Lambda - \Lambda\Lambda$

$\tilde{V}(r)$ [MeV]

$r \ [fm]$

$N\Xi - \Lambda\Lambda$

$V(r)$ [MeV]

$r \ [fm]$

$N\Xi - N\Xi$

$V(r)$ [MeV]

$r \ [fm]$

$\Sigma\Lambda - \Lambda\Lambda$

$V(r)$ [MeV]

$r \ [fm]$

$\Sigma\Xi - \Lambda\Lambda$

$V(r)$ [MeV]

$r \ [fm]$

$\Sigma\Xi - N\Xi$

$V(r)$ [MeV]

$r \ [fm]$

$\Sigma\Xi - \Sigma\Xi$

$V(r)$ [MeV]

$r \ [fm]$

$(96,96) \ src \ \ t−t_0 = 12$

$^3SD_1$

$^1S_0$
$S = -2, \ I = 1, \ BB$ potentials

$(96,96) \ src \ t - t_0 = 12$
single-particle potential $\Xi$
in nuclear matter
single-particle potential $\Xi$
in nuclear matter

spectrum: $e_{\Xi}(k; \rho) = \frac{k^2}{2 M_\Xi} + U_{\Xi}(k; \rho)$

This $U$ is important quantity determining chemical potential of particle in matter
Brueckner-Hartree-Fock

- Single-particle potential of $\Xi$

$$U_\Xi(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle kk' | G^{SLJ}_{(\Xi N)(\Xi N)}(e_\Xi(k) + e_N(k')) | kk' \rangle$$

- $\Xi N$ G-matrix using $M_{N,Y}^{\text{Phys}}, U_{n,p}^{\text{AV18+UIX}}, U_{\Lambda,\Sigma}^{\text{LQCD}}, V_{s=-2}^{\text{LQCD}}, U_{\Xi}^{\text{LQCD}}$

Flavor symmetric $^1S_0$ sectors

- $Q=0$

  $G^{SLJ}_{(\Xi^0 n)(\Xi^0 n)}$, $G_{(\Xi^0 n)(\Xi^- p)}$, $G_{(\Xi^0 n)(\Sigma^+ \Sigma^-)}$, $G_{(\Xi^0 n)(\Sigma^0 \Sigma^0)}$, $G_{(\Xi^0 n)(\Sigma^0 \Lambda)}$, $G_{(\Xi^0 n)(\Lambda \Lambda)}$

- $Q=+1$

  $G^{SLJ}_{(\Xi^0 p)(\Xi^0 p)}$, $G_{(\Xi^0 p)(\Sigma^+ \Lambda)}$

- $Q=-1$

  $G_{(\Xi^0 n)(\Xi^- n)}$, $G_{(\Xi^- n)(\Sigma^- \Lambda)}$
Hyperon single-particle potentials

- QCD leads that $\Xi$ feels attraction $\sim 4$ MeV in SNM
- Results are compatible with experimental suggestion.

$U_\Lambda^{\text{Exp}}(0) \approx -30$, $U_\Xi(0)^{\text{Exp}} \approx -10$?, $U_\Sigma^{\text{Exp}}(0) \geq +20$? [MeV]
# Hyperon single-particle potentials

- Breakdown of $U_Y(\theta; \rho_0)$ in SNM including spin, iso-spin multiplicity

<table>
<thead>
<tr>
<th></th>
<th>$I=1/2$</th>
<th>$I=3/2$</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Λ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^1S_0$</td>
<td>-3.49</td>
<td>-4.97</td>
<td>-28.16</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td>-24.84</td>
<td>21.80</td>
<td></td>
</tr>
<tr>
<td>$^3D_1$</td>
<td>0.18</td>
<td>-0.43</td>
<td></td>
</tr>
<tr>
<td>Σ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^1S_0$</td>
<td>7.43</td>
<td>-4.97</td>
<td>14.62</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td>-9.28</td>
<td>21.80</td>
<td></td>
</tr>
<tr>
<td>$^3D_1$</td>
<td>0.07</td>
<td>-0.43</td>
<td></td>
</tr>
<tr>
<td>Ξ</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^1S_0$</td>
<td>-4.48</td>
<td>9.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td>-4.37</td>
<td>-3.74</td>
<td></td>
</tr>
<tr>
<td>$^3D_1$</td>
<td>-0.01</td>
<td>-0.08</td>
<td></td>
</tr>
</tbody>
</table>

All ΞN S-wave interactions provide attraction in SNM except for $I=1$, $S=0$ channel.

Our prediction based on QCD
Ξ-atom and Ξ-nucleus
$\Xi N$ $G$-matrix potential

- We want to add $^{60}\text{Ni-}\Xi$ strong potential to that calc.
We want to add $^{60}\text{Ni}-\Xi$ strong potential to that calc.

We employ the “G-matrix potential” which is a local potential made so that simulate on-shell $G_{\Xi N}^{SLJ}(k,k)$

$$\frac{2}{\pi} \int_0^\infty r^2 j_L(kr) \tilde{G}_{\Xi N}^{SLJ}(r) j_L(kr) \approx G_{\Xi N}^{SLJ}(k,k)$$

$$\tilde{G}_{\Xi N}^{SLJ}(r) = C_1 e^{-b_1 r^2} + C_2 e^{-b_2 r^2} + C_3 e^{-b_3 r^2}$$

We want to add $^{60}\text{Ni-}\Xi$ strong potential to that calc.

We employ the “G-matrix potential” which is a local potential made so that simulate on-shell $G_{\Xi N}^{SLJ}(k, k)$.

$$\frac{2}{\pi} \int_0^\infty r^2 j_L(k r) \tilde{G}^{SLJ}(r) j_L(k r) \approx G^{SLJ}(k, k)$$

$$\tilde{G}^{SLJ}(r) = C_1 e^{-b_1 r^2} + C_2 e^{-b_2 r^2} + C_3 e^{-b_3 r^2}$$

We neglect the imaginary part of $G$ this time.

$\Xi N$ $G$-matrix potential in SNM
\( \Xi N \) \textit{G}-matrix potential in SNM

\begin{itemize}
  \item \( \Xi N \text{ I}=0, ^1S_0 \)
  \begin{itemize}
    \item \( 0.00\rho_0 \), green
    \item \( 0.25\rho_0 \), blue
    \item \( 0.50\rho_0 \), red
    \item \( 0.75\rho_0 \), pink
    \item \( 1.00\rho_0 \), black
    \item \( V_{11} \), dashed
  \end{itemize}

  \textbf{ρ dependence is not simple}

  \item \( \Xi N \text{ I}=1, ^1S_0 \)
  \begin{itemize}
    \item \( 0.00\rho_0 \), green
    \item \( 0.25\rho_0 \), blue
    \item \( 0.50\rho_0 \), red
    \item \( 0.75\rho_0 \), pink
    \item \( 1.00\rho_0 \), black
    \item \( V_{11} \), dashed
  \end{itemize}

  \textbf{ρ dependence is not simple}

  \item \( \Xi N \text{ I}=0, ^3S_1 \)
  \begin{itemize}
    \item \( 0.00\rho_0 \), green
    \item \( 0.25\rho_0 \), blue
    \item \( 0.50\rho_0 \), red
    \item \( 0.75\rho_0 \), pink
    \item \( 1.00\rho_0 \), black
    \item \( V_{11} \), dashed
  \end{itemize}

  \item \( \Xi N \text{ I}=1, ^3S_1 \)
  \begin{itemize}
    \item \( 0.00\rho_0 \), green
    \item \( 0.25\rho_0 \), blue
    \item \( 0.50\rho_0 \), red
    \item \( 0.75\rho_0 \), pink
    \item \( 1.00\rho_0 \), black
    \item \( V_{11} \), dashed
  \end{itemize}

\end{itemize}
$^{60}\text{Ni-Ξ}$ strong potential

- Chage base, spin averaged ΞN G-matrix potential at 0.5$\rho_0$
**$^{60}\text{Ni-Ξ} \text{ strong potential}**

- **Chage base, spin averaged $\Xi N$ G-matrix potential at $0.5\rho_0$**
- **Single folding potential with $^{60}\text{Ni}$ proton/neutron density**

\[
U(\vec{r})_{\text{Ni-Ξ}} = \int d^3\vec{r}' \rho_{\text{Ni}}^p(\vec{r}') \, U_{\Xi p}(|\vec{r} - \vec{r}'|; \tilde{\rho}) + \int d^3\vec{r}' \rho_{\text{Ni}}^n(\vec{r}') \, U_{\Xi n}(|\vec{r} - \vec{r}'|; \tilde{\rho})
\]

- **Fixed $\tilde{\rho} = 0.5 \rho_0$ is used, for the moment, because of the complicated $\rho$ dependence of $U(r; \rho)$, i.e. very primitive.**
We see downward energy shifts due to the QCD $\Xi N$.
We see downward energy shifts due to the QCD $\Xi N$

- But,...
  - S-wave spin-averaged $U_{\Xi N}(r)$ may not be enough.
  - Imaginary potential $W_{\Xi N}(r)$ would be important.
  - Only shifts in larger $L$ states are accessible in experiments...
    - eg. X-ray of transition from $L=7$ bottom to $L=6$ bottom
- We need to improve our calculation more.
Summary and outlook

☆ We have all BB S-wave interaction from QCD
  • Lattice QCD simulation at a almost physical point
  • HAL QCD method ie. potential method

☆ Especially, we have a theoretical ΞN interaction

☆ Our QCD ΞN interaction leads
  • attraction of Ξ in SNM at ρ₀ about UΞ(0) = −4 MeV
  • which seems qualitatively consistent with experiments
  • All S-wave ΞN interaction provide attraction except for I=1,S=0

☆ We’ studied Ξ-atom to check our QCD ΞN
  • G-matrix potential + Folding = ⁶⁰Ni-Ξ strong potential
  • We obtained downward energy shifts as expected
  • In addition, we found a nuclear bound state with 5 MeV binging

☆ But, we have to improve our calculation
  • Include imaginary part of G-matrix potential
  • Folding including density dependence of G-matrix pot.
  • Try very tiny shifts of larger L states, L=6,7….
Thank you !!
Backup
FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?
FAQ

1. Does your potential depend on the choice of source?
   ➔ No. Some sources may enhance excited states in 4-point func. However, it is no longer a problem in our new method.

2. Does your potential depend on choice of operator at sink?
   ➔ Yes. It can be regarded as the “scheme” to define a potential. Note that a potential itself is not physical observable. We will obtain unique result for physical observables irrespective to the choice, as long as the potential $U(r,r')$ is deduced exactly.
3. Does your potential $U(r,r')$ or $V(r)$ depends on energy?

> By definition, $U(r,r')$ is non-local but energy independent. While, determination and validity of its leading term $V(r)$ depend on energy because of the truncation.

However, we know that the dependence in $NN$ case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.} = 0 – 90$ MeV. We rely on this in our study.

If we find some dependence, we will determine the next leading term of the expansion from the dependence.
4. Is the H a compact \textit{six-quark} object or a tight \textit{BB} bound state?
FAQ

found in $SU(3)^F$ limit, ie. heavy u,d quark world

4. Is the H a compact six-quark object or a tight $BB$ bound state?

→ Both.
There is no distinct difference between two in QCD. Note that baryon is made of three quarks in QCD. Imagine a compact 6-quark object in $(0S)^6$ configuration. This configuration can be re-written in a form of $(0S)^3 \times (0S)^3 \times \text{Exp}(-a \ r^2)$ with relative coordinate $r$. This demonstrate that a compact six-quark object, at the same time, has a $BB$ type configuration. In LQCD simulation at $SU(3)^F$ limits, we've established existence of a $B=2, S=-2, I=0$ stable QCD eigenstate.