What can we learn about twist-2 GPDs through quasi-distributions?

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Based on:
Outline

➢ Definition of (Quasi-) GPDs

➢ Analytical Results of Quasi-GPDs in Scalar Diquark Model

➢ Numerical Results in Scalar Diquark Model
  • Choice of parameters
  • Quasi-PDFs
  • Quasi-GPDs

➢ Moments of Quasi-GPDs
  • First moment
  • Second moment & Ji’s spin-sum rule

➢ Summary
Definition of (Quasi-) GPDs

Light-cone (standard) correlator \(-1 \leq x \leq 1\)

\[
F^\Gamma [x, \Delta; \lambda, \lambda'] = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik \cdot z} \times \langle p'; \lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma W(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) | p; \lambda \rangle \bigg|_{z^+=z_-=0}
\]

- **Time dependence:**
  \[ z^0 = \frac{1}{\sqrt{2}} (z^+ + z^-) = \frac{1}{\sqrt{2}} z^- \]
- **Cannot** be computed on Euclidean lattice
- **Parameterization:** \( \gamma^+ \rightarrow H \quad E \)
  \( \gamma^+ \gamma_5 \rightarrow \tilde{H} \quad \tilde{E} \)
  \( i\sigma^j \gamma_5 \rightarrow H_T \quad E_T \quad \tilde{H}_T \quad \tilde{E}_T \)
- **Kinematical variables:**
  \[
x = \frac{k^+}{P^+} \quad \xi = -\frac{\Delta^+}{2P^+} \quad t = \Delta^2 = -\frac{(4\xi^2 M^2 + \Delta_{\perp}^2)}{1 - \xi^2}
\]

Correlator for quasi-GPDs (Ji, 2013) \(-\infty \leq x \leq \infty\)

\[
F_Q^\Gamma (x, \Delta; \lambda, \lambda'; P^3) = \frac{1}{2} \int \frac{dz^3}{2\pi} e^{ik \cdot z} \times \langle p', \lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma W_Q(-\frac{z}{2}, \frac{z}{2}) \psi(\frac{z}{2}) | p, \lambda \rangle \bigg|_{z^0=\tilde{z}_\perp=0}
\]

- **Non-local correlator depending on position** \(z^3\)
- **Can** be computed on Euclidean lattice
- **Parameterization:** \( \gamma^{0/3} \rightarrow H_{Q(0/3)} \quad E_{Q(0/3)} \)
  \( \gamma^{0/3} \gamma_5 \rightarrow \tilde{H}_{Q(0/3)} \quad \tilde{E}_{Q(0/3)} \)
  \( i\sigma^j \gamma_5 \rightarrow H_{T,Q(0/3)} \quad E_{T,Q(0/3)} \)
- **Kinematical variables:**
  \[
x = \frac{k^3}{P^3} \neq \frac{k^+}{P^+} \quad \xi \quad |\Delta_{\perp}| \quad P^3
\]
Analytical Results of Quasi-GPDs in Scalar Diquark Model

Correlator for quasi-GPDs:

\[
F_Q^{[\Gamma]}(x, \Delta; P^3) = \frac{ig^2}{2(2\pi)^4} \int dk^0 d^2k_\perp \frac{\bar{u}(p') \left( \frac{k + \Delta}{2} + m_q \right) \Gamma \left( \frac{k - \Delta}{2} + m_q \right) u(p)}{D_{GPD}}
\]

\[
D_{GPD} = \left[ \left( k + \frac{\Delta}{2} \right)^2 - m_q^2 + i\varepsilon \right] \left[ \left( k - \frac{\Delta}{2} \right)^2 - m_q^2 + i\varepsilon \right] \left[ (P - k)^2 - m_s^2 + i\varepsilon \right]
\]

Quasi-GPDs: Example:

\[
H_{Q(0)}(x, \xi, t; P^3) = \frac{ig^2 P^3}{(2\pi)^4} \int dk^0 d^2k_\perp \frac{N_{H(0)}}{D_{GPD}}
\]

\[
N_{H(0)} = \delta (k^0)^2 - \frac{2}{P^3} \left[ x \left( P^3 \right)^2 - m_q M - x \frac{t}{4} - \frac{1}{2} \delta \xi t \frac{k_\perp \cdot \vec{A}_\perp}{\vec{A}_\perp^2} \right] k^0 + \delta \left[ x^2 \left( P^3 \right)^2 + k_\perp^2 + m_q^2 + (1 - 2x) \frac{t}{4} - \delta \xi t \frac{k_\perp \cdot \vec{A}_\perp}{\vec{A}_\perp^2} \right]
\]

- Standard (twist-2) GPDs are continuous in entire \( x \) range
- Position of \( k^0 \)-poles do not depend on \( x \) - Quasi-GPDs are continuous at \( x = \pm \xi \)
- For \( P^3 \to \infty \), we recover ALL 8 leading-twist standard GPDs in SDM
Numerical Results in Scalar Diquark Model

Choice of parameters:

- **Nucleon-Quark-Diquark coupling**: \( g = 1 \)

- **Masses**: **Constraint** - \( M < m_q + m_s \) \((M - 
  \text{nucleon mass, } m_q - \text{quark mass, } m_s - \text{spectator mass})\)

  **Values assumed** - \( M = 0.939 \text{ GeV} \), \( m_q = 0.35 \text{ GeV} \), \( m_s = 0.70 \text{ GeV} \) (Gamberg, Kang, Vitev, Xing, 2014)

- **Cut-off for \(|k_\perp|\) integration**: \( \Lambda = 1 \text{ GeV} \)

- **Momentum transfer**: \( |\tilde{\Delta}_\perp| = 0 \text{ GeV} \)

Variations of these parameters do not affect our general results
Quasi-PDFs:

- Example: Unpolarized quasi-PDF

- General features of quasi helicity & transversity distributions are same as that of unpolarized quasi-PDF

- For larger $P^3$, good agreement between quasi & standard PDFs for a wide range of $x$

- Considerable discrepancies at small $x$ & large $x$

Small - $x$ issue

- Standard PDFs are discontinuous at $x = 0$
  Example: $f_1(x < 0) = 0$

- Quasi-PDFs are continuous for all $x$

- Quasi-PDFs must agree with standard PDFs & therefore must rapidly change around $x = 0$
**Large - $x$ issue**

- Observing this issue through – Relative Difference

  **Example:**
  \[ R_{f_1(0)}(x; P^3) = \frac{f_1(x) - f_{1,Q(0)}(x; P^3)}{f_1(x)} \]

- Cut-graph model analysis:

  \[
  \tilde{x} = x + \frac{1}{4(P^3)^2} \left( \frac{k_1^2 + m_s^2}{1 - x} - (1 - x) M^2 \right) + O \left( \frac{1}{(P^3)^4} \right)
  \]

a) Mismatch in this region between $x = \frac{k^+}{P^+}$ & $\tilde{x} = \frac{k_3}{P^3}$ for finite $P^3$

b) Better results at very large $x$ on relating $\tilde{x}$ & $x$

**Example:** $f_{1,Q}$
Quasi-GPDs:

- **Example**: Unpolarized quasi-GPDs
  - Large - $x$ issue persists for ALL quasi-GPDs
  - Large - $x$ situation worsens if $\xi$ increases (more severe for $\tilde{E}_Q$, $\tilde{E}_{T,Q}$)
**Quasi-GPDs in ERBL region**

**Example**: Unpolarized quasi-GPDs

- Large discrepancies between ALL quasi-GPDs & standard distributions in ERBL region if $\xi$ is small (compare with region around $x = 0$ for PDFs)

- Lattice QCD calculations might be promising if $\xi$ is larger
Exploring different skewness variables

- Standard & quasi-skewness variables: Model-independent relations

\[
\begin{align*}
\xi &= - \frac{\Delta^+}{2P^+} \\
\tilde{\xi}_3 &= - \frac{\Delta^3}{2P^3} = \delta \xi \\
\tilde{\xi}_0 &= - \frac{\Delta^0}{2P^0} = \frac{\xi}{\delta} \\
\delta &= \sqrt{1 + \frac{M^2 - t/4}{(P^3)^2}}
\end{align*}
\]

- Significant discrepancies between variables for finite \( P^3 \)

- Comparative study of quasi-GPDs as functions of \( \xi \) vs. quasi-GPDs as functions of \( \tilde{\xi}_3 \) & \( \tilde{\xi}_0 \) :
Moments of Quasi-GPDs

**Link between Quasi-GPDs & Form Factors:**

\[
(P^3)^{n+1} \int_{-\infty}^{\infty} dx x^n \int_{-\infty}^{\infty} \frac{dz^3}{2\pi} e^{ixP^3z^3} \langle p', \lambda' | \bar{\psi}^q(-\frac{z^3}{2}) \Gamma W_Q(-\frac{z}{2}, \frac{z}{2}) \psi^q(\frac{z^3}{2}) | p, \lambda \rangle \bigg|_{z^0=z_-=0} = \langle p', \lambda' | \bar{\psi}^q(0) \Gamma (\frac{i}{2} \tilde{D}_3)^n \psi^q(0) | p, \lambda \rangle
\]

**First moment:** \( n = 0 \)

\[
\Gamma = \gamma^0 \quad \Gamma = \gamma^3
\]

\[
\int_{-1}^{1} dx H^q(x, \xi, t) = \int_{-\infty}^{\infty} dx \left( \frac{1}{\delta} \right) H_{Q(0)}^q(x, \xi, t; P^3) = \int_{-\infty}^{\infty} dx H_{Q(3)}^q(x, \xi, t; P^3) = F_1^q(t)
\]

**Dirac F.F.**

\[
\int_{-1}^{1} dx E^q(x, \xi, t) = \int_{-\infty}^{\infty} dx \left( \frac{1}{\delta} \right) E_{Q(0)}^q(x, \xi, t; P^3) = \int_{-\infty}^{\infty} dx E_{Q(3)}^q(x, \xi, t; P^3) = F_2^q(t)
\]

**Pauli F.F.**

\[
\delta = \sqrt{1 + \frac{M^2 - t/4}{(P^3)^2}}
\]

- Re-define (half) the quasi-GPDs by \( \frac{1}{\delta} \) to restore \( P^3 \)-independence of lowest moments
First moments: $n = 0$

\[
\Gamma = \gamma^0 \gamma_5 \quad \Gamma = \gamma^3 \gamma_5
\]

Axial-vector F.F.

\[
\Gamma = \gamma^\mu \gamma_5
\]

Pseudo-scalar F.F.

\[
\delta = \sqrt{1 + \frac{M^2 - t/4}{(P^3)^2}}
\]

\[
\Gamma = i\sigma^{j0} \gamma_5 \quad \Gamma = i\sigma^{j3} \gamma_5
\]

F.F. of local tensor current

\[
\Gamma = i\sigma^{\mu\nu} \gamma_5
\]

- First moments of quasi-PDFs can be extracted from these relations
Second moment: \( n = 1 \)

- Focus on local vector operator: \( \bar{\psi}^q(0)\gamma^\mu\psi^q(0) \)
- Ji’s spin-sum rule: \( \int_{-1}^{1} dx \, x \left( H^q(x, \xi, t) + E^q(x, \xi, t) \right) = A^q(t) + B^q(t) \)
- Relation between quasi-GPDs & Ji’s spin-sum rule:

\[
\begin{align*}
\int_{-\infty}^{\infty} dx \, x \frac{1}{\delta} \left( H_{Q(0)}^q(x, \xi, t; P^3) + E_{Q(0)}^q(x, \xi, t; P^3) \right) &= \frac{1}{2} (1 + \delta^2)(A^q(t) + B^q(t)) - \frac{1}{2} (1 - \delta^2) D^q(t) \\
\int_{-\infty}^{\infty} dx \, x \left( H_{Q(3)}^q(x, \xi, t; P^3) + E_{Q(3)}^q(x, \xi, t; P^3) \right) &= A^q(t) + B^q(t)
\end{align*}
\]

\[\delta = \sqrt{1 + \frac{M^2 - t/4}{(P^3)^2}}\]

a) Higher-twist contamination in second moment of \( H_{Q(0)} + E_{Q(0)} \); also F.F. from anti-symmetric EMT, \( D \), contributes at finite \( P^3 \)

b) Second moment of \( H_{Q(3)} + E_{Q(3)} \) is related to total angular momentum of quarks
• Second moment relations for PDFs:

\[
\int_{-1}^{1} dx x f_1(x) = A^q(0)
\]

\[
\int_{-\infty}^{\infty} dx x \frac{1}{\delta_0} f_{1,Q(0)}(x; P^3) = A^q(0)
\]

\[
\int_{-\infty}^{\infty} dx x f_{1,Q(3)}(x; P^3) = A^q(0) - \frac{M^2}{(P^3)^2} C^q(0)
\]

a) Second moment of \( f_{1,Q(0)} \) is \( P^3 \)-independent only if divided by \( \delta_0 \)

\[
\delta_0 = \delta(t = 0) = \sqrt{1 + \frac{M^2}{(P^3)^2}}
\]

b) Second moment of \( f_{1,Q(3)} \) is \( P^3 \)-dependent involving F.F. \( \tilde{C} \)

• Our model calculations agree with all moment relations

• \( P^3 \)-dependence of moments either absent or calculable; moments may assist in studying systematic errors in lattice QCD results
Summary

Quasi-GPDs in a scalar diquark model

• For $P^3 \to \infty$, all our expressions for quasi-GPDs agree with 8 leading-twist standard GPDs

• For finite $P^3$, large discrepancies between quasi & standard GPDs at large $x$ (situation worsens with increase in skewness; significance of this feature depends upon GPD)

• For finite $P^3$ & large $\xi$, good agreement between quasi & standard GPDs in ERBL region

• Explored higher-twist effects associated with $x$ & $\xi$

• Model-independent analysis of moments of quasi-distributions (including relation to Ji’s spin-sum rule)

• Moments might help to study systematic uncertainties in lattice QCD
Explicit parameterization equations:

\[
F^{[\gamma^0]}(x, \Delta; \lambda, \lambda'; P^3) = \frac{1}{2P^0} \bar{u}(p', \lambda') \left[ \gamma^0 H_{Q(0)}(x, \xi, t; P^3) + \frac{i\sigma^0 \mu}{2M} E_{Q(0)}(x, \xi, t; P^3) \right] u(p, \lambda)
\]

\[
F^{[\gamma^3 \gamma_5]}(x, \Delta; \lambda, \lambda'; P^3) = \frac{1}{2P^0} \bar{u}(p', \lambda') \left[ \gamma^3 \gamma_5 \tilde{H}_{Q(3)}(x, \xi, t; P^3) + \frac{\Delta^3 \gamma_5}{2M} \tilde{E}_{Q(3)}(x, \xi, t; P^3) \right] u(p, \lambda)
\]

\[
F^{[i\sigma^{30} \gamma_5]}(x, \Delta; \lambda, \lambda'; P^3) = -\frac{i\varepsilon^{03ij}}{2P^0} \bar{u}(p', \lambda') \left[ i\sigma^{3i} H_{T,Q(0)}(x, \xi, t; P^3) + \frac{\gamma^3 \Delta^i_{\perp} - \Delta^3 \gamma^i_{\perp}}{2M} E_{T,Q(0)}(x, \xi, t; P^3) \right.
\]

\[
+ \frac{P^3 \Delta^i_{\perp}}{M^2} \tilde{H}_{T,Q(0)}(x, \xi, t; P^3) - \frac{P^3 \gamma^i_{\perp}}{M} \tilde{E}_{T,Q(0)}(x, \xi, t; P^3) \right] u(p, \lambda)
\]

Parameterizations consistent with forward limit.
Quasi-PDFs:

- For larger $P^3$, not much of a difference between two definitions of quasi-PDFs (& quasi-GPDs):

- Plots for polarized quasi-PDFs (specific Dirac structures):

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Helicity

Transversity
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Polarized Quasi-GPDs:

- Plots for **Longitudinally Polarized Quasi-GPDs** (specific Dirac structures):

- Plots for **Transversely Polarized Quasi-GPDs** (specific Dirac structures):
Sensitivity of numerical results to model parameters:

- **Variation of quark mass \( m_q \)**
  - \( P^3 = 2 \text{ GeV} \) \( m_q = 1.0 \text{ GeV} \)
  - \( m_q = 0.01 \text{ GeV} \)
  - \( m_q = 0.10 \text{ GeV} \)
  - \( m_q = 0.35 \text{ GeV} \)

- **Variation of spectator mass \( m_s \)**
  - \( P^3 = 2 \text{ GeV} \) \( m_q = 0.35 \text{ GeV} \)

- **Variation of cut-off \( \Lambda \)**
  - \( P^3 = 2 \text{ GeV} \) \( m_s = 0.7 \text{ GeV} \) \( m_q = 0.35 \text{ GeV} \)
  - \( \Lambda = 1 \text{ GeV} \)
  - \( \Lambda = 4 \text{ GeV} \)

- **Variation of transverse momentum transfer \( \vec{\Delta}_\perp \)**
  - \( \Delta_\perp = 0 \text{ GeV} \) \( \xi = 0.1 \)
  - \( m_s = 0.7 \text{ GeV} \)
  - \( m_q = 0.35 \text{ GeV} \)

- **Variation of quark mass - mild impact on R.D. for ALL PDFs & GPDs (\* no pattern for ERBL)** Except: \( h_1 \) at small \( x \)

- **Greater sensitivity to variation in \( m_s \) for ALL PDFs & GPDs (\*); Smaller values of \( m_s \) is optimal**

- **Mild impact of variations in \( \Lambda \)**
  - Exception(s): functions changing sign

- **Mild impact of variations in \( \vec{\Delta}_\perp \) on R.D. for ALL GPDs**
Most general local EMT: (C. Lorcé, H. Moutarde, A. P. Trawiński, EPJ C, 2019)

\[
\langle p', \lambda' | T^{\mu\nu, q}(0) | p, \lambda \rangle = \bar{u}(p', \lambda') \left[ \frac{P^\mu P^\nu}{M} A^q(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} C^q(t) + M g^{\mu\nu} \bar{C}^q(t) + \frac{P^{[\mu} i \sigma^{\nu]}}{4M} \Delta^\alpha (A^q(t) + B^q(t)) + \frac{P^{[\mu} i \sigma^{\nu]} \Delta^\alpha}{4M} D^q(t) \right] u(p, \lambda)
\]