Process-independent effective charge: from QCD's Green's functions to Hadron phenomenology

Daniele Binosi,
Cedric Mezrag,
Joannis Papavassiliou,
Craig D. Roberts,
José Rodríguez-Quintero,
K. Raya, J. Segovia, F. de Soto ...

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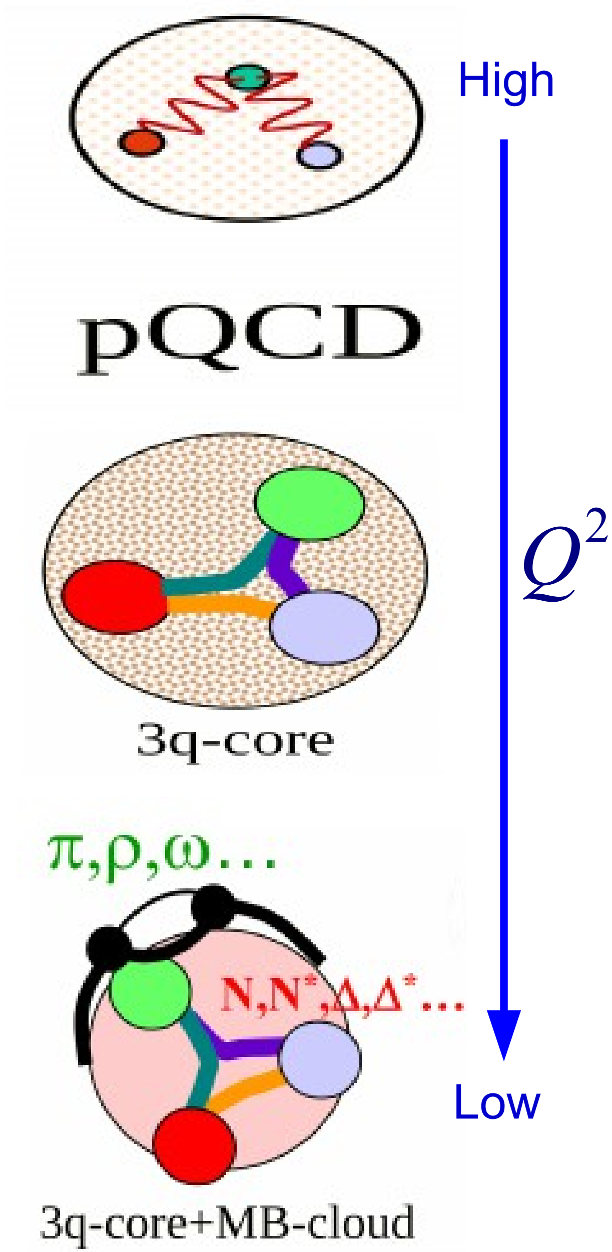
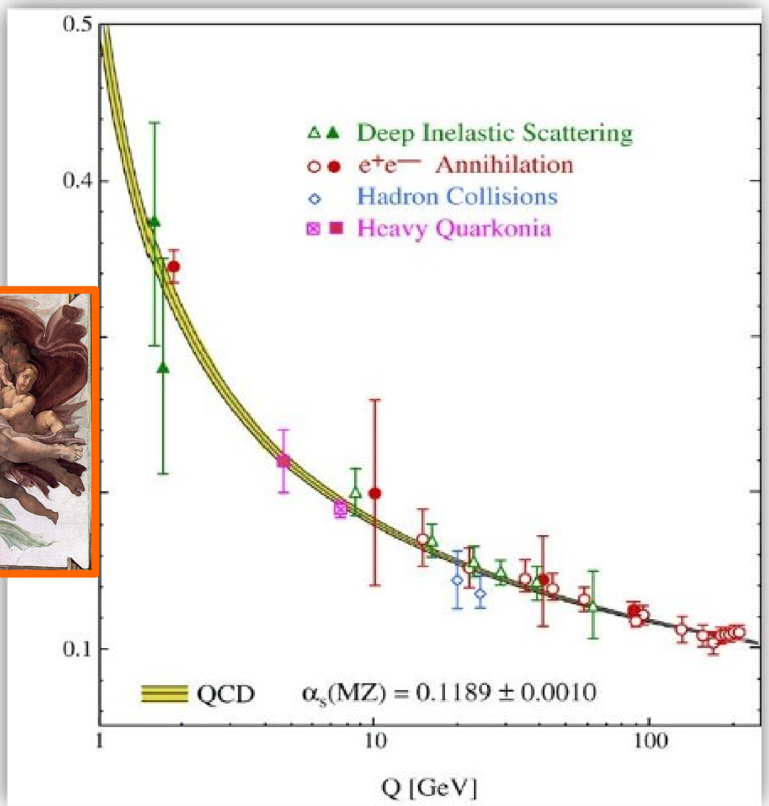
[Few Body Syst. 59 (2018) no.6, 121]
Preliminary results...

INPC 2019; SEC, Glasgow; July 28th- August 2nd

QCD's running coupling. Motivation.



The QCD Holy Grail: the understanding of hadrons in terms of its elementary excitations; namely, quarks and gluons!



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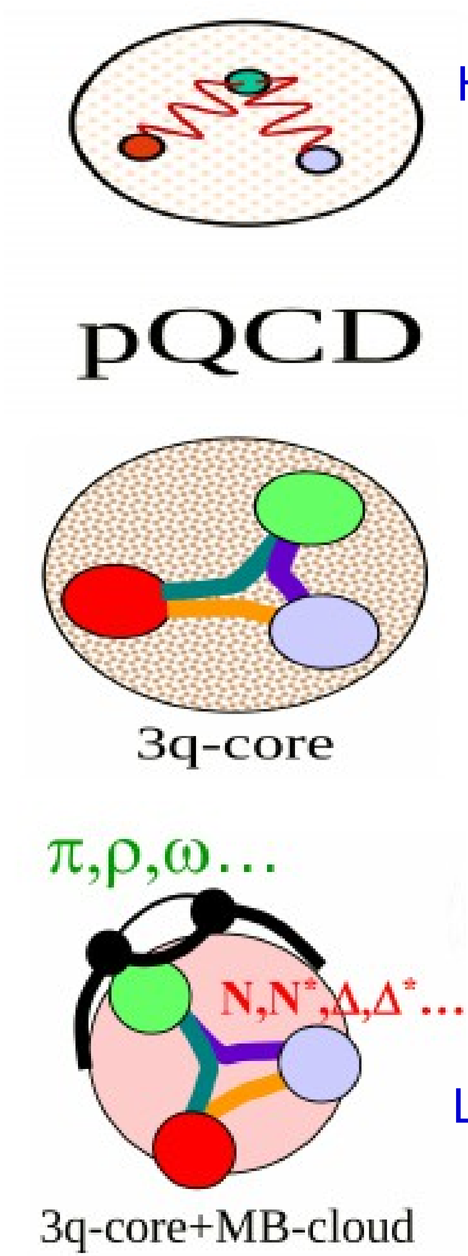


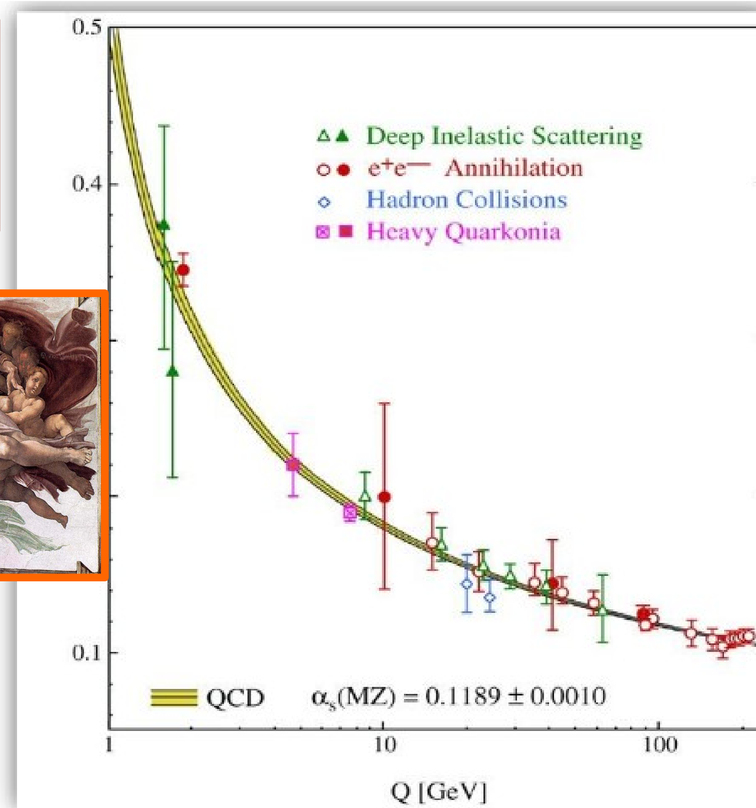
High

Q^2

Low

Confinement

Colored bound states have never been seen to exist as particles in nature

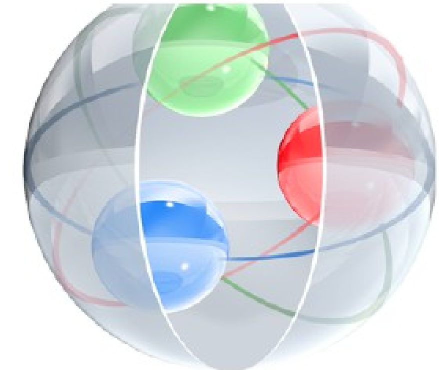


DCSB

Chiral symmetry appears dynamically violated in the Hadron spectrum

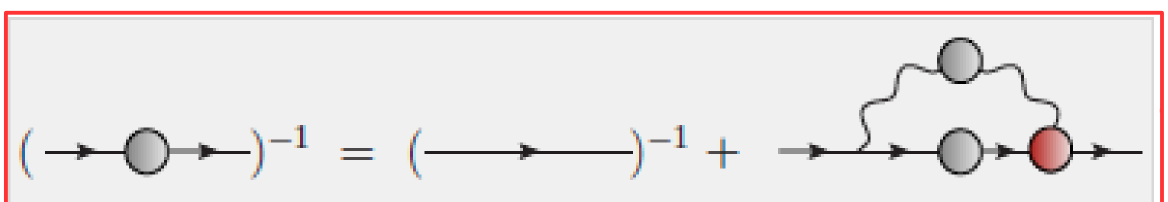
Emergent phenomena playing a dominant role in the real world dominated by the IR dynamics of QCD.

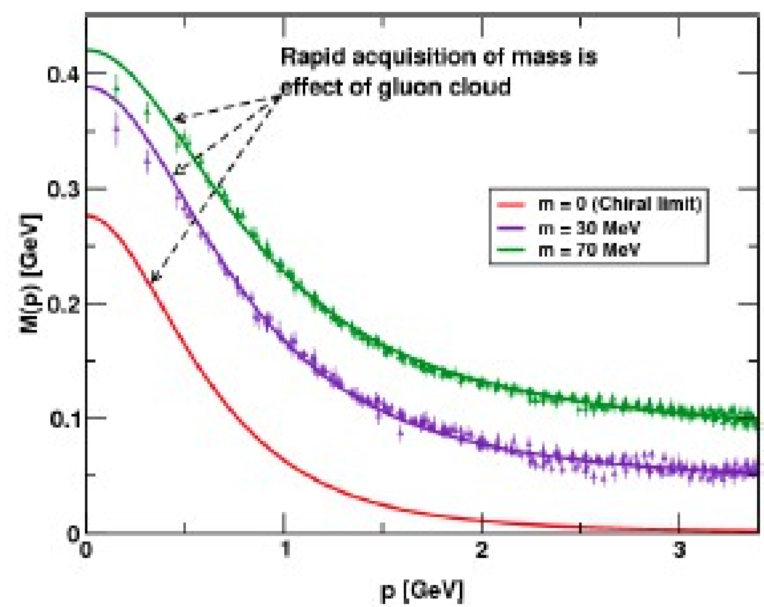
Quark's gap equation



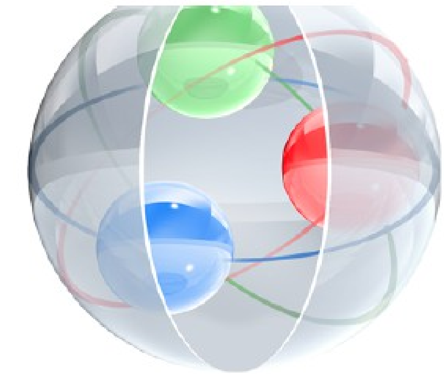
Let's start by the beginning:

- Dynamical chiral symmetry breaking generates the “constituent-quark” masses
- This is the most important mechanism for the mass generation in our Universe (responsible for around 98 % of the proton mass)
- The effect is realized through the pure theory's dynamics through the quark's gap equation and is clearly achieved through the pure theory's dynamics (nothing needs to be added to QCD!!)



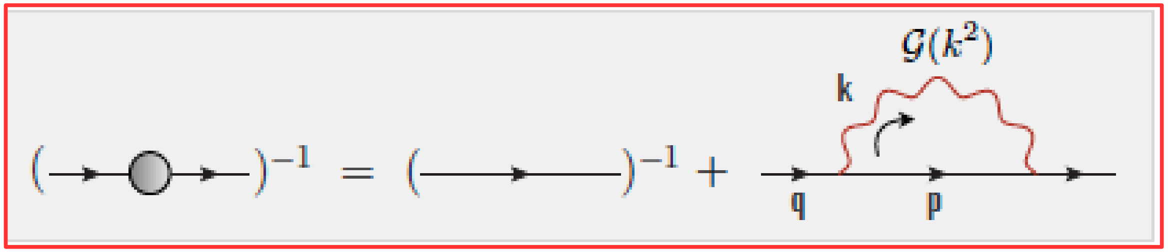


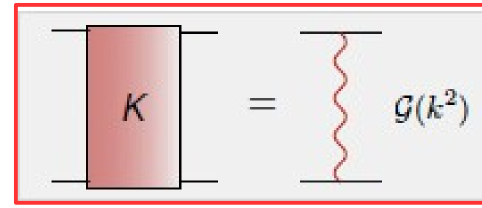
Quark's gap equation



Beyond Rainbow-Ladder truncation:

One-gluon exchange effective kernel + ~~Tree-level~~ quark-gluon vertex





$$S^{-1}(p) = Z_2\,(i\gamma\cdot p + m^{\rm bm}) + \Sigma(p)\,,$$

$$\Sigma(p) = Z_1 \int_{dq}^{\Lambda} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\Gamma_\nu(q,p),$$

$$4\pi\, I(k^2)\frac{1}{k^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$\mathcal{I}(k^2) = k^2\frac{\mathcal{G}_{\rm IR}(k^2)+\mathcal{G}_{\rm UV}(k^2)}{4\pi}$$

$$\mathcal{G}_{\rm IR}(k^2) = \frac{8\pi^2}{\omega^5}\varsigma^3 e^{-k^2/\omega^2}$$

$$\mathcal{G}_{\rm UV}(k^2) = \frac{96\pi^2}{25}\frac{1-e^{-k^2/1[\rm GeV^2]}}{k^2\log[e^2-1+(1+k^2/\Lambda^2)^2]}$$

- Ball-Chiu vertex [PRD(22)1980]
- Anomalous Chromomagnetic vertex

Consistent with both linear and transverse STI

$$\Gamma_\mu = \Gamma_\mu^{BC} + \Gamma_\mu^{ACM}$$

Model parameters:

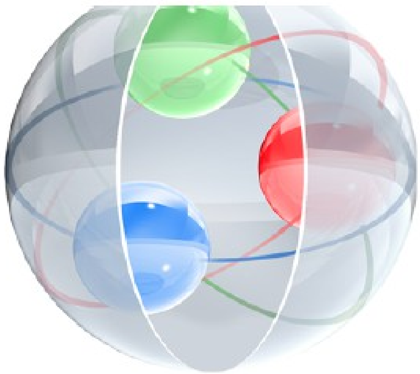
$\Lambda = 0.234\ GeV$

$\zeta = 0.55\ GeV$

$\omega \in [0.4, 0.6]\ GeV$

Fixed by the pion decay constant

Quark's gap equation



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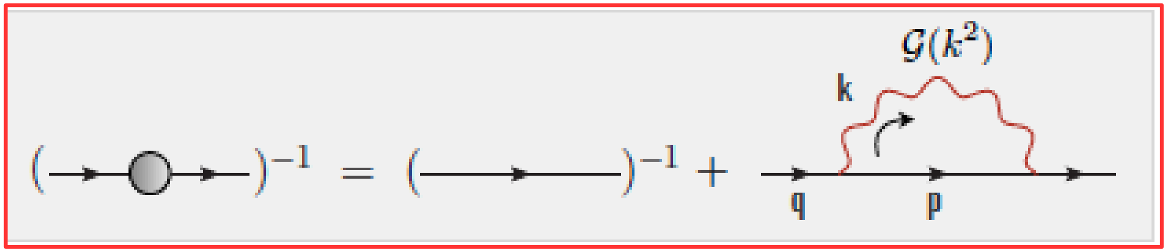
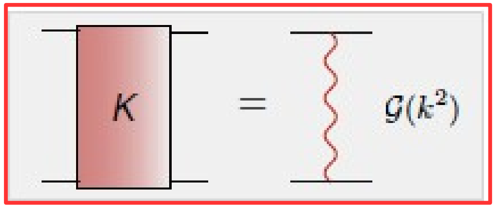
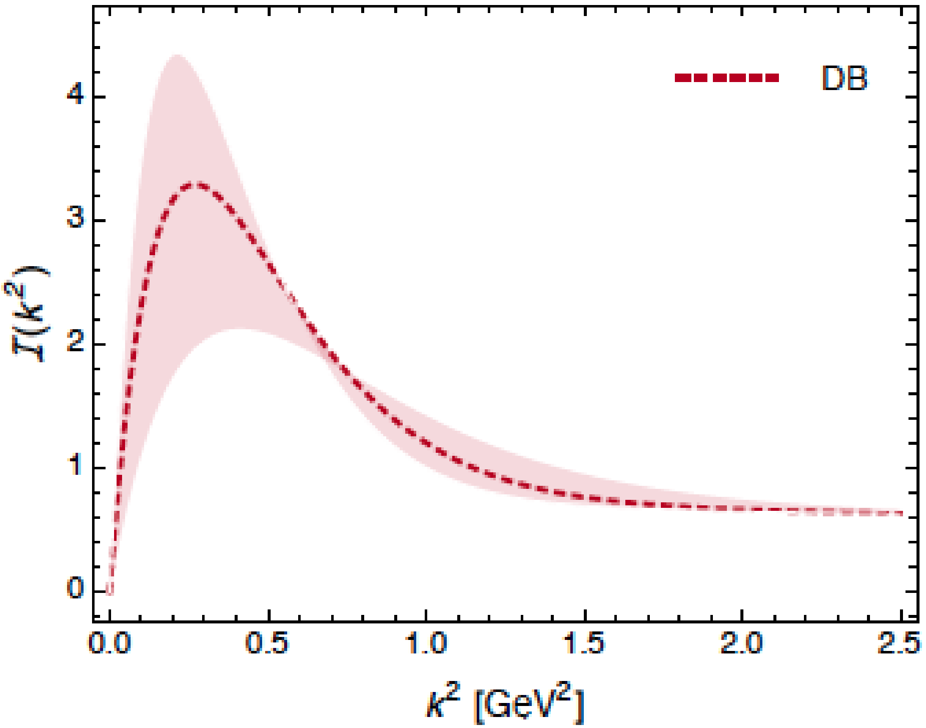
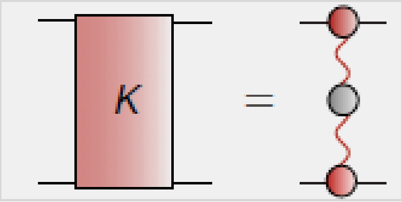






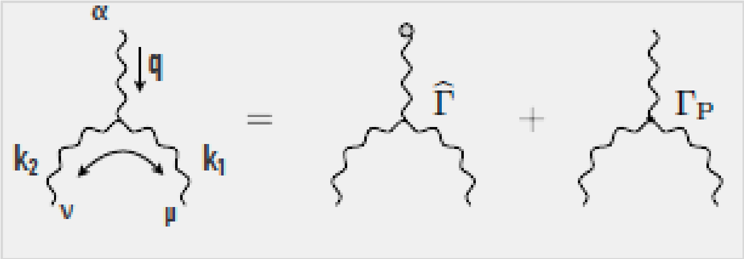
Quark's gap equation: RGI interaction



- **Universal (process-independent) contribution:**
  originates entirely from the gauge sector

  Fundamental quantities: PT-BFM propagators/vertices
  satisfy Abelian-like Slavnov-Taylor (ST) identities

  How to get them?
  use the PT algorithm
  Cornwall, Papavassiliou, PRD 40 (1989)

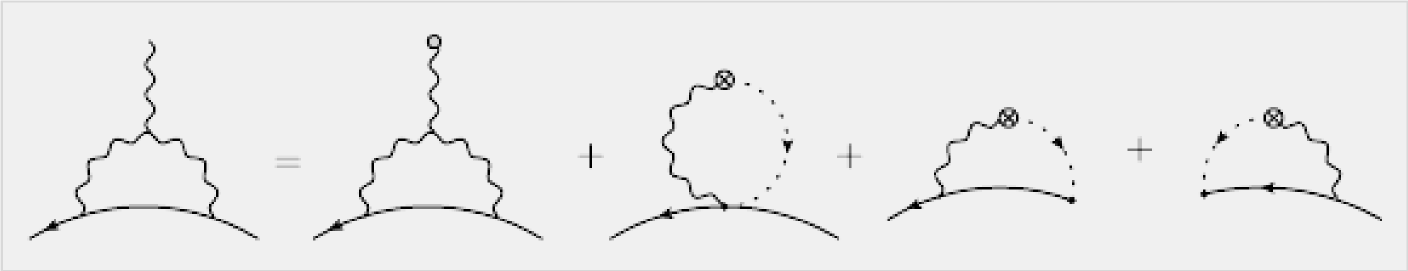


$$\widehat{\Gamma}^{\alpha\mu\nu} = (k_2 - k_1)^\alpha g^{\mu\nu} + 2q^\nu g^{\alpha\mu} - 2q^\mu g^{\alpha\nu}$$

$$\Gamma_P^{\alpha\mu\nu} = k_1^\mu g^{\alpha\nu} - k_2^\nu g^{\alpha\mu}$$

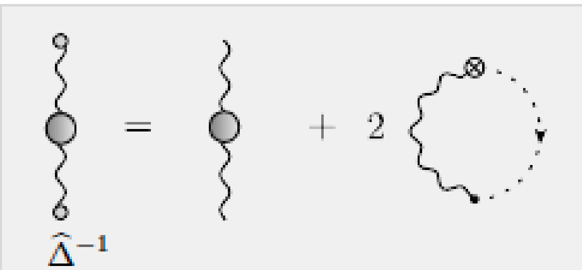
- **longitudinal momenta**
  trigger elementary Ward identities

Apply the PT to the quark-gluon vertex
one loop result:

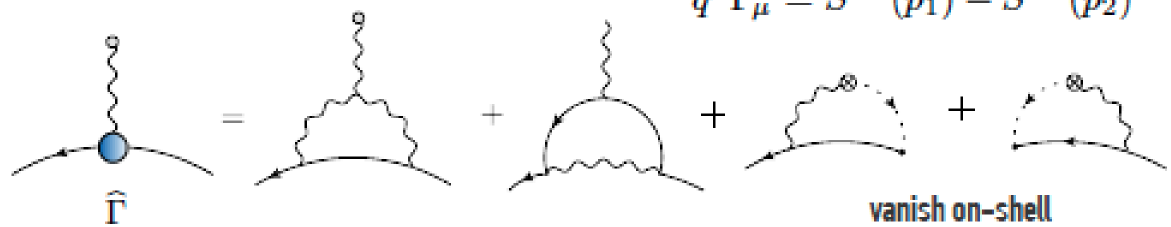


Quark's gap equation: RGI interaction

- **Allot pieces to different Green's functions**
  construct $\widehat{\Delta}$ and $\widehat{\Gamma}_\mu$



$q^\mu \widehat{\Gamma}_\mu = S^{-1}(p_1) - S^{-1}(p_2)$



- **Crucial all-order equivalence: PT=BFM**
  yields Feynman rules for systematic calculation

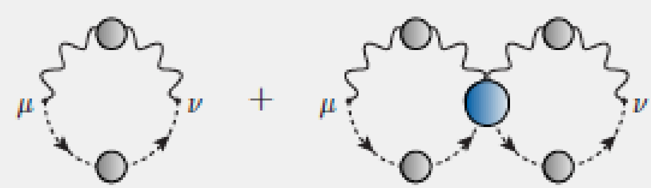
$$\widehat{\Delta} \sim \frac{1}{q^2[1 + bg^2 \log q^2/\mu^2]}; \qquad b = 11C_A/48\pi^2$$

- **Absorbs all the RG logs**
  as the photon in QED
- **Renormalizes as Z_g^{-2}**

- **An additional equivalence holds: antiBRST+BRST=BFM**
  plethora of symmetry identities, in particular BQ identities
  DB, Quadri, PRD 88 (2013)

$$\Delta(q^2) = [1 + G(q^2)]^2 \widehat{\Delta}(q^2)$$

$\Lambda_{\mu\nu}(q) =$



$$= G(q^2) g_{\mu\nu} + L(q^2) \frac{q_\mu q_\nu}{q^2}$$

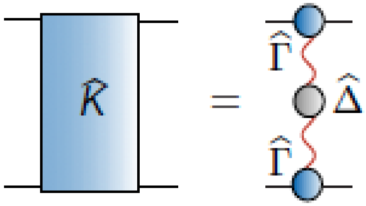
- ***G* special PT-BFM function:**
  determined by ghost-gluon dynamics
- **Combination 1+*G* appears in *all* BQIs**
  fundamental non-Abelian quantity
- ***G* is related (Landau gauge) to the ghost dressing:**
  use ghost gap equation to constrain *1+G, L*

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2)$$

Quark's gap equation: RGI interaction

Convert vertices/propagators into PT-BFM ones
new RG invariant combination appears

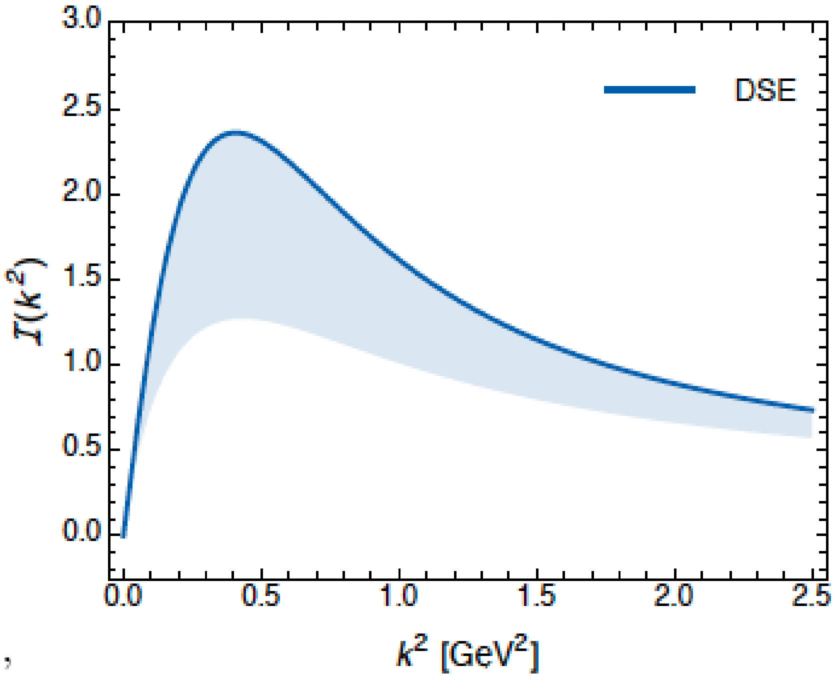
$$\widehat{d}(k^2) = \alpha(\mu^2)\widehat{\Delta}(k^2;\mu^2)$$



Use symmetry identity
to identify the interaction strength
Aguilar, DB, Papavassiliou, Rodriguez-Quintero, PRD 90 (2009)
DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

$$\mathcal{I}(k^2) = k^2\widehat{d}(k^2)$$

$$\widehat{d}(k^2) = \frac{\alpha(\mu^2)\Delta(k^2;\mu^2)}{[1+G(k^2;\mu^2)]^2}$$



1+*G* and *L* determined by their own SDEs
under simplifying assumptions:

$$1+G(p^2)=Z_c - g^2\int_k \left[2+\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2},$$

$$L(p^2)=-g^2\int_k \left[1-4\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2}.$$

$$F^{-1}(q^2) \;=\; Z_c - 3\,g^2\int_k \left[1-\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2}$$

- **Main source of uncertainties:**
  needs assumptions on ghost vertex behavior
- **Parametrized by δ∈[0,1]**
  lower bound (δ=0): *1/F=1+G*

Quark's gap equation: RGI interaction

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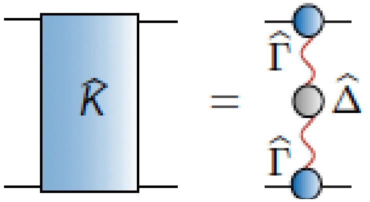

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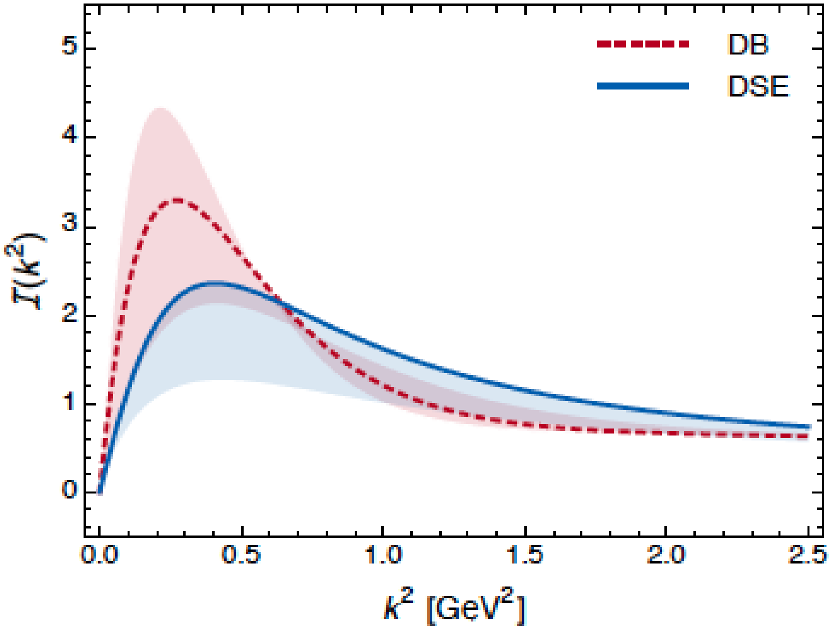

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- **Main source of uncertainties:** needs assumptions on ghost vertex behavior
- **Parametrized by δ∈[0,1]** lower bound (δ=0): *1/F=1+G*

Both top-bottom and down-up approaches deliver quark-gluon effective interactions which compare remarkably well with each other!

QCD effective charge

Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

$$F(k^2;\zeta^2) \underset{k^2/\Lambda_{\rm T}^2 \gg 1}{\approx} \left(\ln \frac{k^2}{\Lambda_{\rm T}^2} \Big/ \ln \frac{\zeta^2}{\Lambda_{\rm T}^2} \right)^{-\widetilde{\gamma}_0/\beta_0},$$

$$L(k^2;\zeta^2) \underset{k^2/\Lambda_{\rm T}^2 \gg 1}{\approx} \frac{3g^2(\zeta^2)}{32\pi^2} \left(\ln \frac{k^2}{\Lambda_{\rm T}^2} \Big/ \ln \frac{\zeta^2}{\Lambda_{\rm T}^2} \right)^{-(\widetilde{\gamma}_0+\gamma_0)/\beta_0}$$

$$L(k^2)F(k^2) \underset{q^2/\Lambda_{\rm T}^2 \gg 1}{\approx} \frac{3}{2\beta_0 \ln(k^2/\Lambda_{\rm T}^2)},$$

$$\alpha_{\overline{\rm MS}}(k^2)(1 + 1.09\, \alpha_{\overline{\rm MS}}(k^2) + \ldots)$$

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$$I(k^2) \underset{k^2/\Lambda_{\rm T}^2 \ll 1}{\approx} k^2 \widehat{d}(0) \left[1 - \left(\frac{\widehat{d}(0)}{8\pi} + \frac{f_w}{m_g^2} \right) k^2 \ln \frac{k^2}{\Lambda_{\rm T}^2} \right]$$

$$L(k^2)F(k^2) \underset{q^2/\Lambda_{\rm T}^2 \gg 1}{\approx} \frac{3}{2\beta_0 \ln (k^2/\Lambda_{\rm T}^2)},$$

$$\widehat{d}(0) = \frac{\alpha_0}{m_0^2} \approx \frac{0.9\pi}{(m_P/2)^2}$$

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D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLB 742 (2015)

QCD effective charge

Let us first carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

Remarkable QCD feature: saturation of the RG key ingredient $\hat{d}(0)$

$$\widehat{d}(0) = \frac{\alpha_0}{m_0^2} \approx \frac{0.9\pi}{(m_P/2)^2}$$

D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLB 742 (2015)

Define then the RGI invariant function

$$\mathcal{D}(k^2) = \frac{\Delta(k^2;\mu^2)}{\Delta(0;\mu^2) m_0^2}$$

Extract the (process-independent) coupling

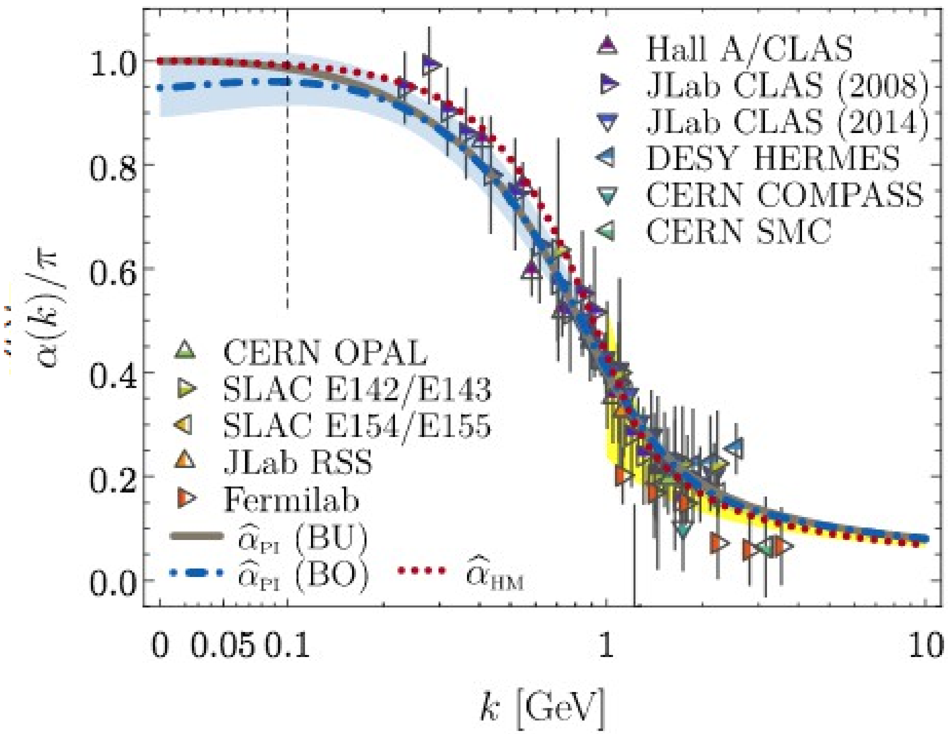
Using the quark gap equation

$$\Sigma(p) = Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{\alpha}_{\rm PI}(k^2) \mathcal{D}_{\mu\nu}(k^2) \gamma_\mu S(q) \hat{\Gamma}^a_\nu(q,p)$$

$$\widehat{\alpha}(k^2) = \frac{\widehat{d}(k^2)}{\mathcal{D}(k^2)} \underset{k^2 \gg m_0^2}{\longrightarrow} \mathcal{I}(k^2)$$

D. Binosi, C. Mezrag, J. Papavassiliou, J.R-Q, C.D. Roberts, arXiv:1612.04835

QCD effective charge: comparison.



- **Equivalence in the perturbative domain**
  reasonable definitions of the charge

$$\alpha_{g_1}(k^2) = \alpha_{\overline{MS}}(k^2)[1 + 1.14\alpha_{\overline{MS}}(k^2) + \cdots]$$
$$\widehat{\alpha}_{PI}(k^2) = \alpha_{\overline{MS}}(k^2)[1 + 1.09\alpha_{\overline{MS}}(k^2) + \cdots]$$

- **Equivalence in the non-perturbative domain**
  highly non-trivial (ghost-gluon interactions)

- **Agreement with light-front holography**
  model for α_{g_1}
  Deur, Brodsky, de Teramond, PPNP 90 (2016)

- **Process dependent effective charges**
  fixed by the leading-order term in the expansion of a given observable
  Grunberg, PRD 29 (1984)

- **Bjorken sum rule**
  defines such a charge
  Bjorken, PR 148 (1966); PRD 1 (1970)

$$\int_0^1 dx\,[g_1^p(x,k^2) - g_1^n(x,k^2)] = \frac{g_A}{6}[1 - \alpha_{g_1}(k^2)/\pi]$$

  - $g_1^{p,n}$ **spin dependent p/n structure functions**
    extracted from measurements using unpolarized targets
  - g^A **nucleon flavour-singlet axial charge**

- **Many merits**
  - **Existence of data**
    for a wide momentum range
  - **Tight sum rules constraints on the integral**
    at IR and UV extremes
  - **Isospin non-singlet**
    suppress contributions from hard-to-compute processes

D. Binosi, C. Mezrag, J. Papavassiliou, J.R-Q, C.D. Roberts, arXiv:1612.04835

PI-effective charge from lattice data with Nf=3 flavors at the physical point

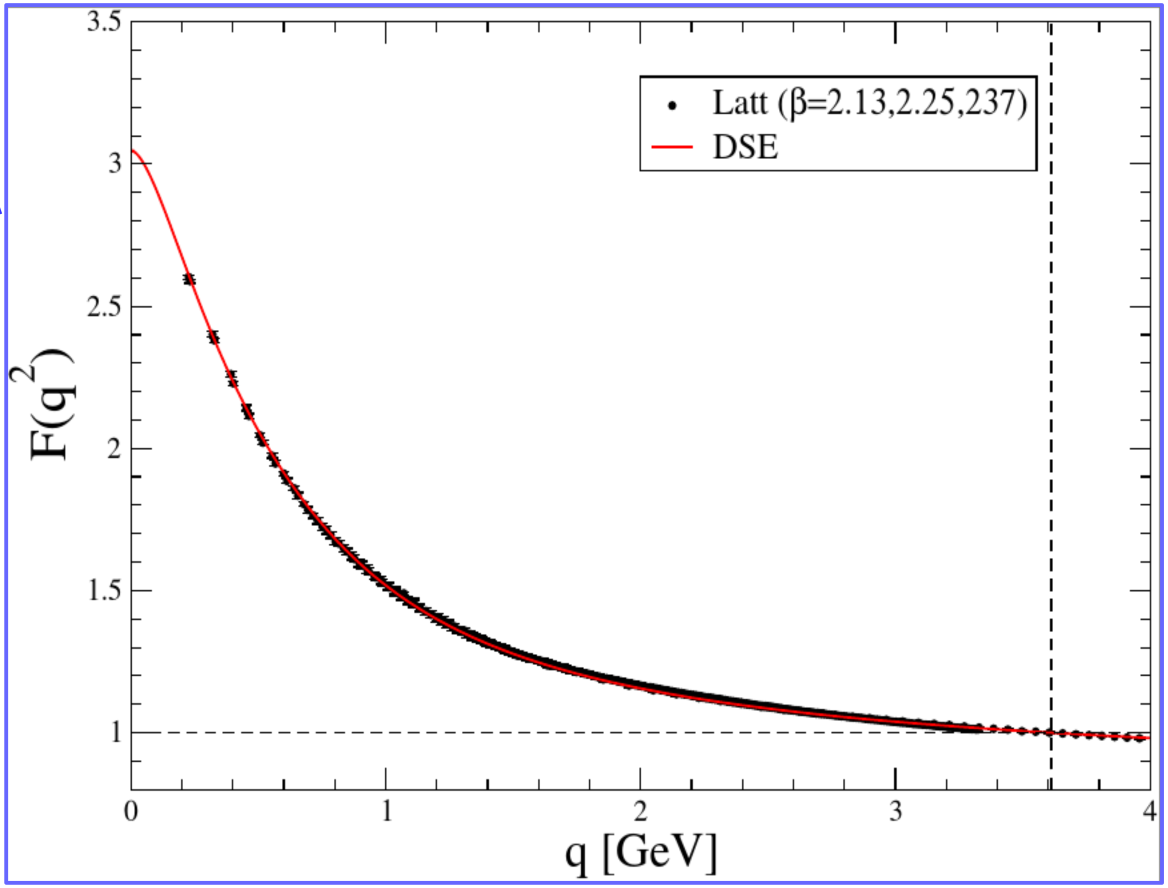
Preliminary results:

$$\widehat{\alpha}_{\rm PI}(q^2) = \frac{\widehat{d}(q^2)}{\mathcal{D}(q^2)} \simeq \frac{\alpha_T(q^2)}{q^2\left[1 - L(q^2,\zeta^2)F(q^2,\zeta^2)\right]^2}\,\frac{m_0^2\Delta_F(0,\zeta^2)}{\Delta_F(q^2,\zeta^2)}$$

$$= \alpha_T(\zeta^2)\,\frac{F(q^2,\zeta^2)}{\left[1 - L(q^2,\zeta^2)F(q^2,\zeta^2)\right]^2}\,\Delta_F(0,\zeta^2)m_0^2$$

The IR running of the PI effective charge with momenta only depends on:

- The ghost dressing function



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The IR running of the PI effective charge with momenta only depends on:

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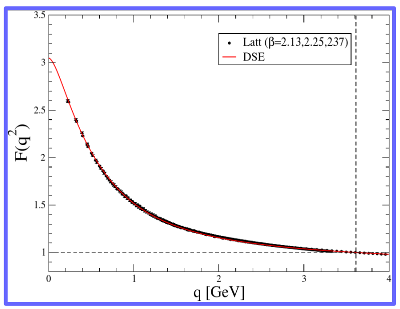
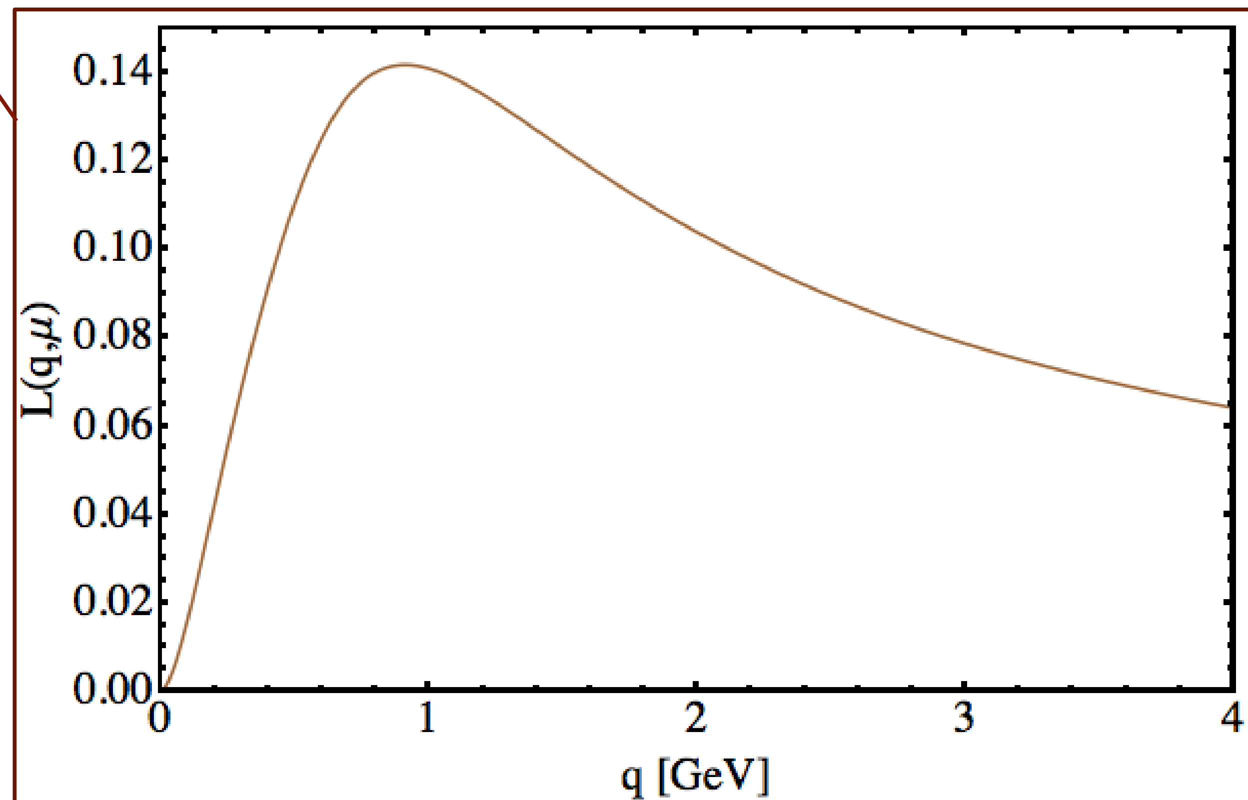
- The PT-BFM function L





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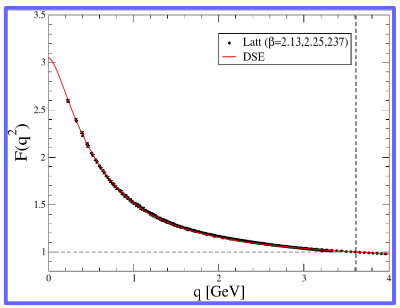
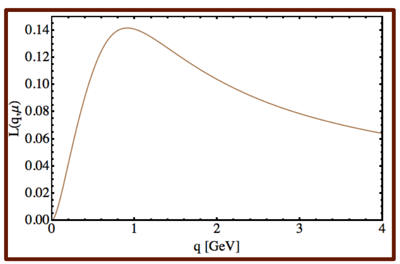
$$= \alpha_T(\zeta^2)\,\frac{F(q^2,\zeta^2)}{[1 - L(q^2,\zeta^2)F(q^2,\zeta^2)]^{\,2}}\,\Delta_F(0,\zeta^2)m_0^2$$

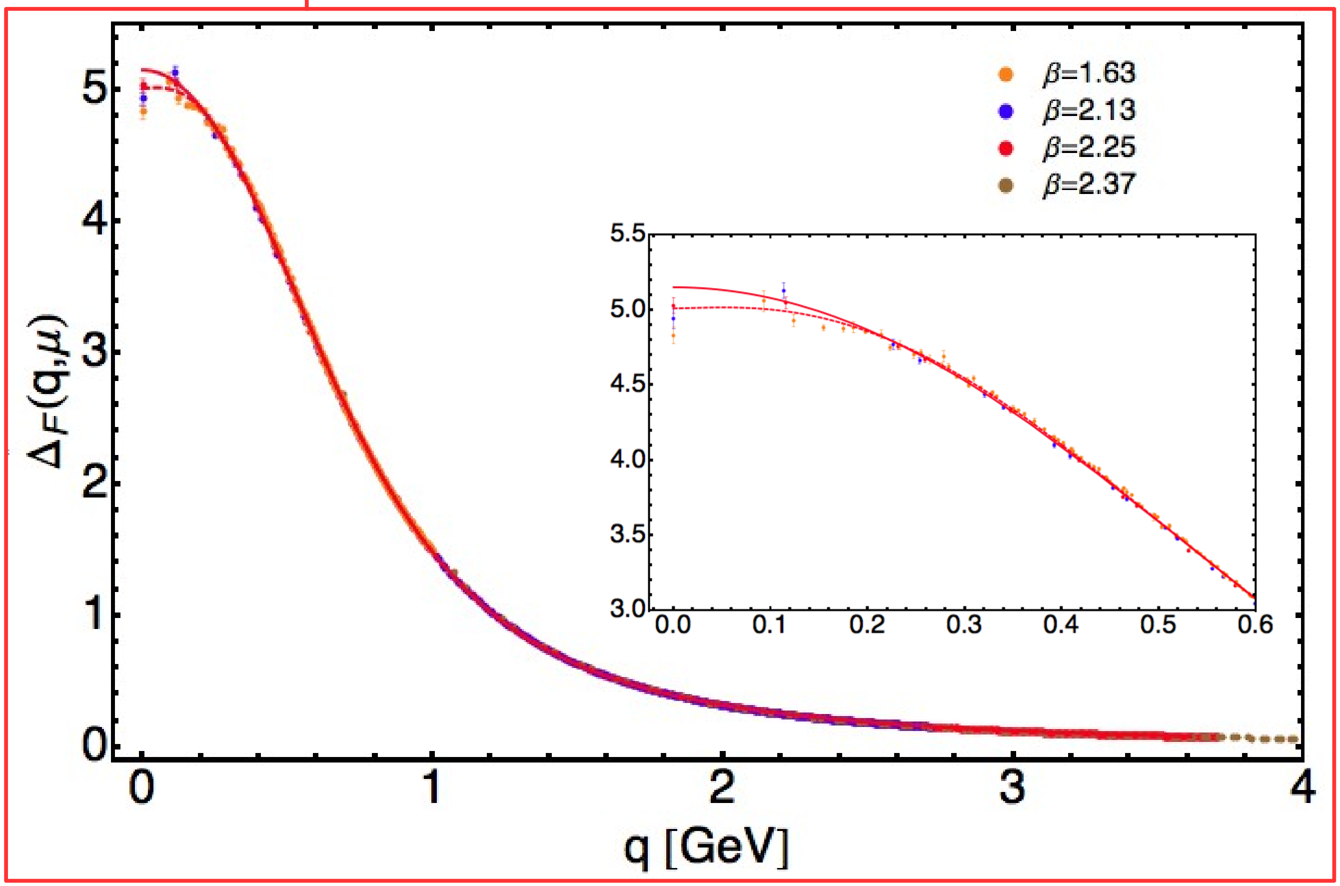
The IR running of the PI effective charge with momenta only depends on:

- The ghost dressing function
- The PT-BFM function L

Its strength depends also on the saturation point at zero-momentum of the gluon propagator







PI-effective charge from lattice data with Nf=3 flavors at the physical point

Preliminary results:

$$\widehat{\alpha}_{\rm PI}(q^2) = \frac{\widehat{d}(q^2)}{\mathcal{D}(q^2)} \simeq \frac{\alpha_T(q^2)}{q^2\left[1-L(q^2,\zeta^2)F(q^2,\zeta^2)\right]^2}\,\frac{m_0^2\Delta_F(0,\zeta^2)}{\Delta_F(q^2,\zeta^2)}$$

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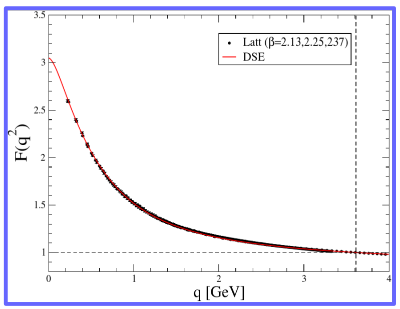
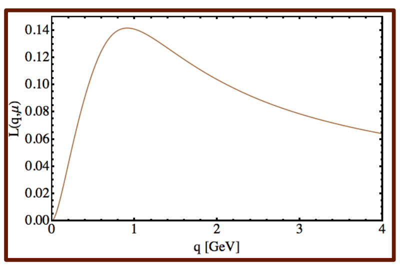
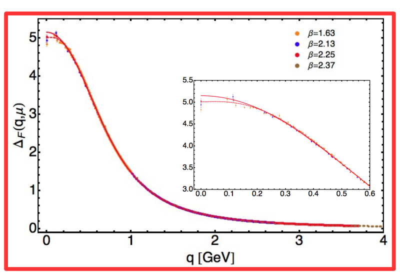
The IR running of the PI effective charge with momenta only depends on:

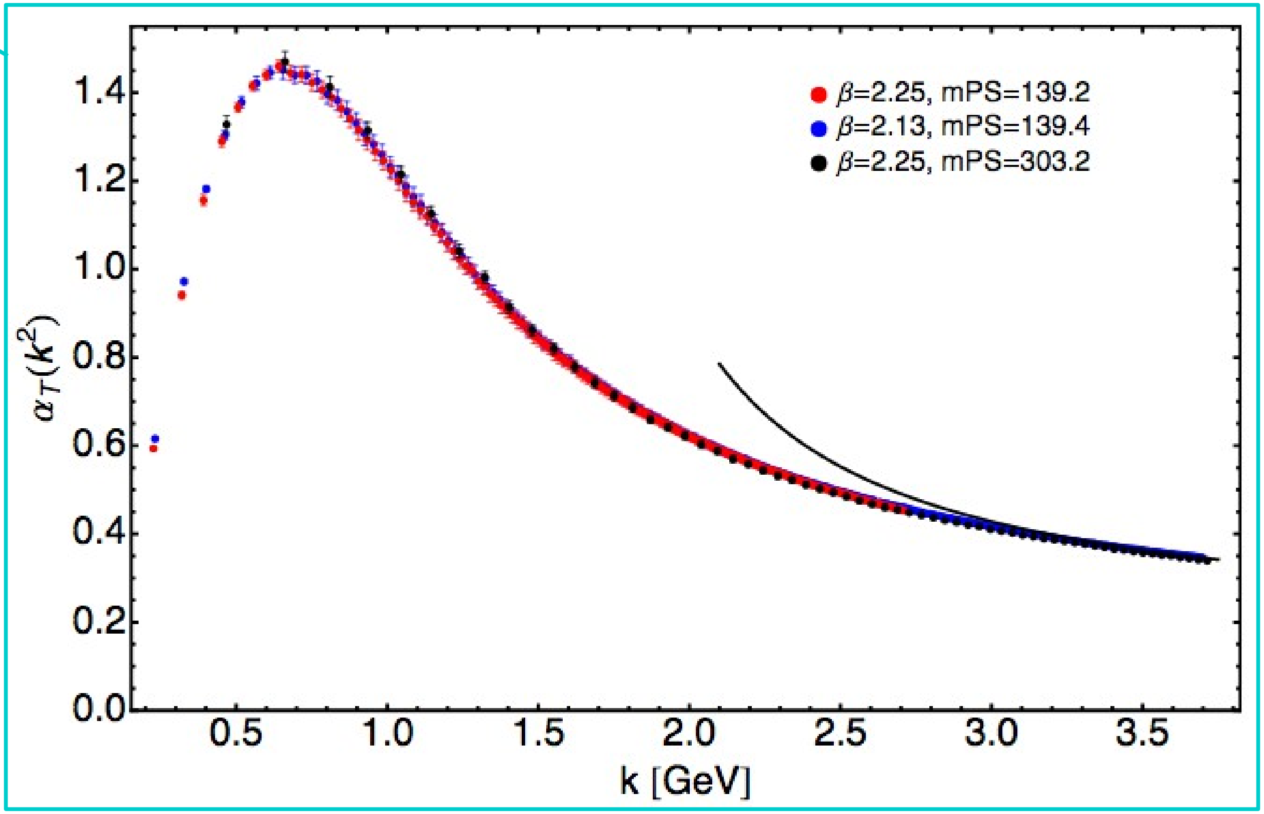
- The ghost dressing function
- The PT-BFM function L

Its strength depends also on the saturation point at zero-momentum of the gluon propagator and on the Taylor coupling.









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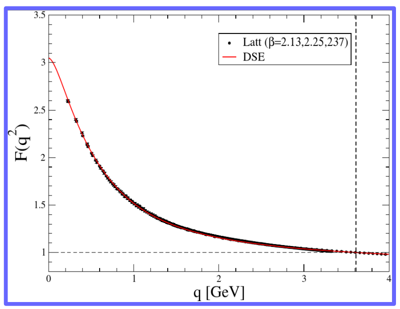
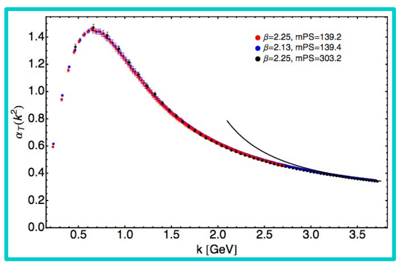
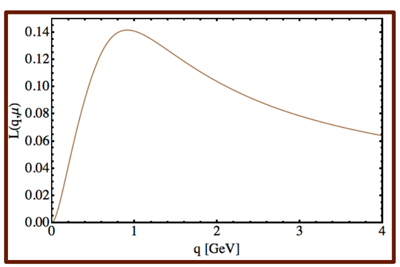
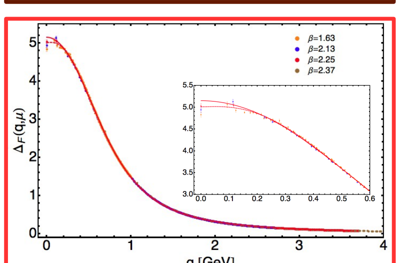


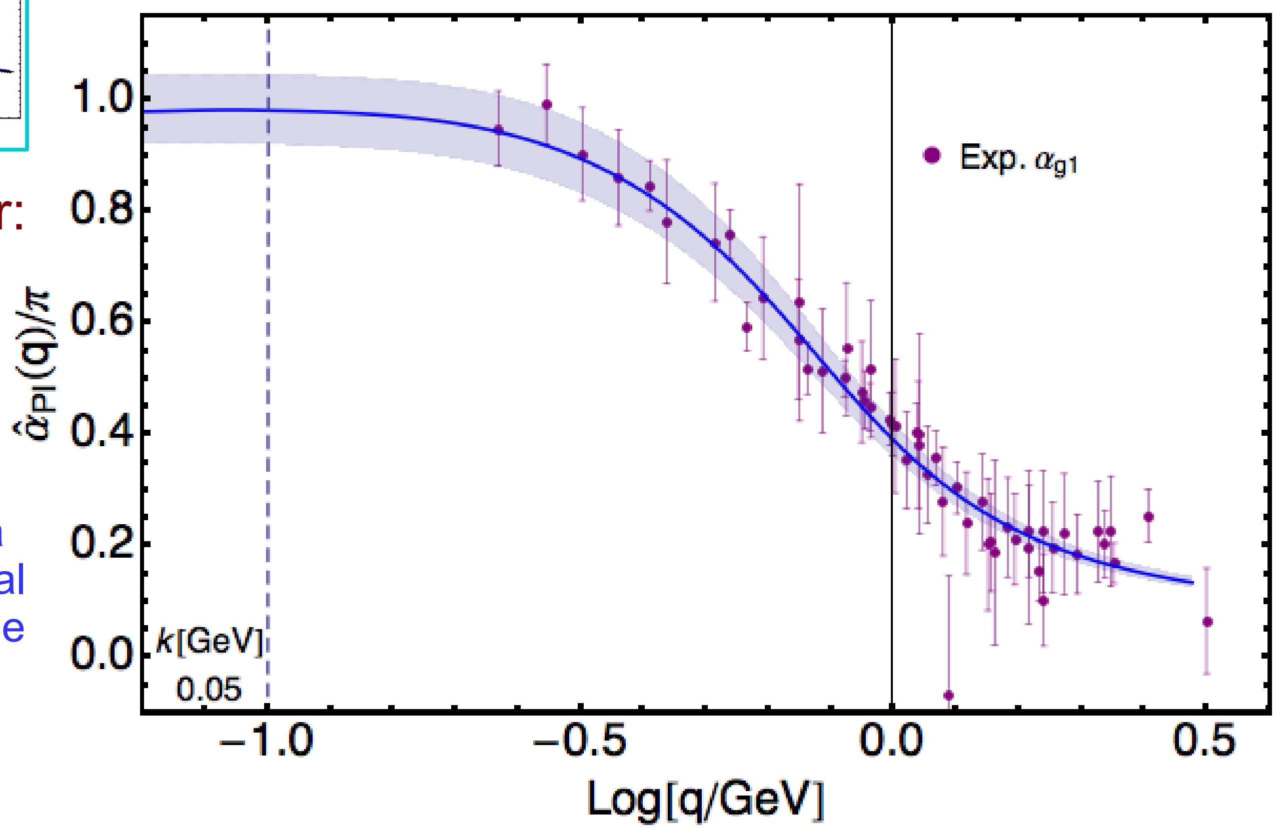






All put together:

Less uncertainties (that of the gluon mass is only left here) and still a better agreement with the world data for the experimental determination of the Bjorken sum-rule effective charge.



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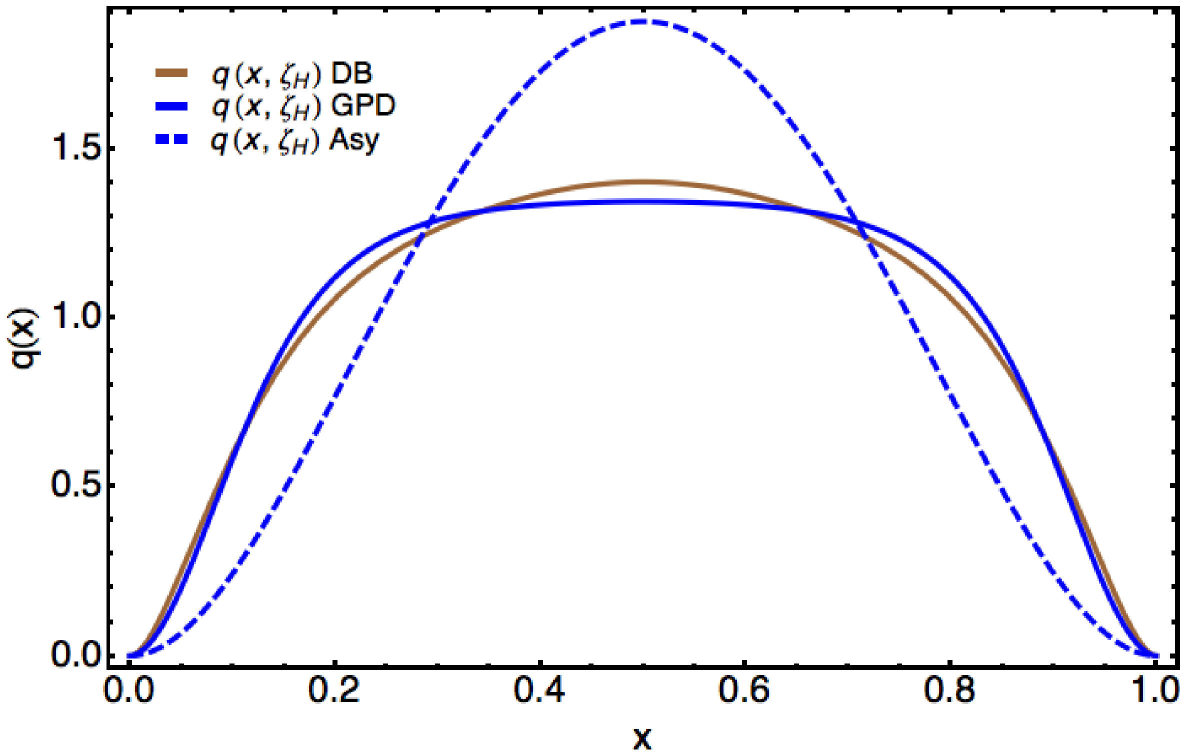

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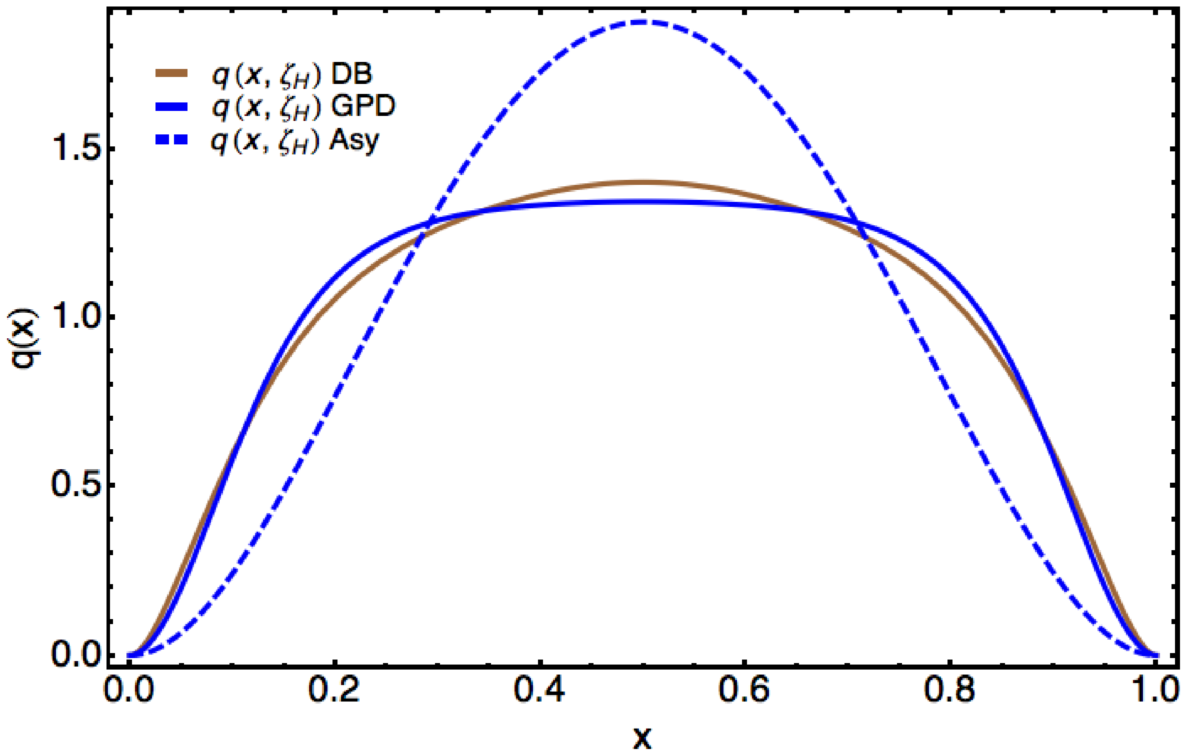

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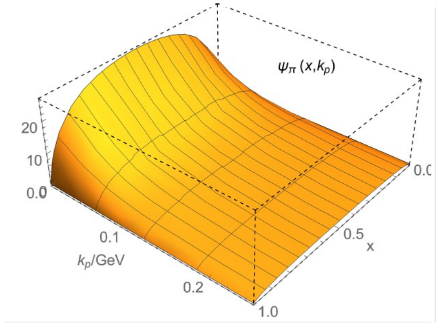
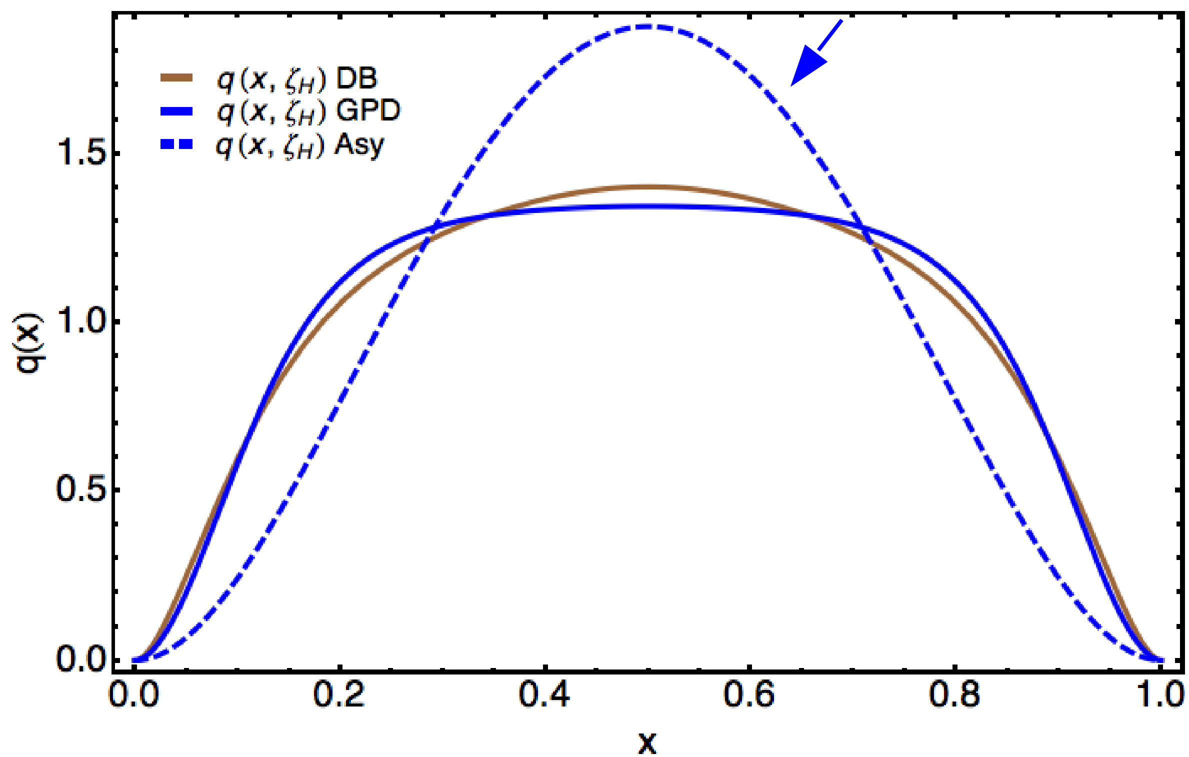


LFWF leading to asymptotic PDAs

$$q_{\rm sf}(x) \approx 30\, x^2(1-x)^2$$



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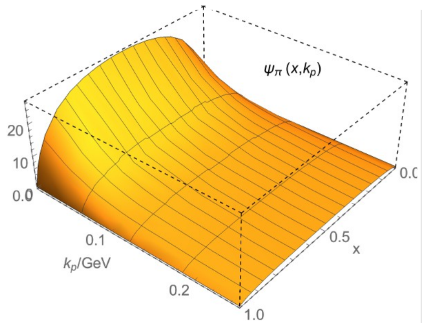

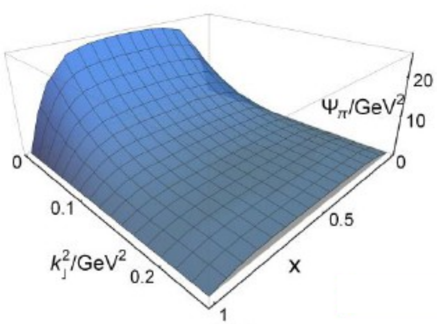

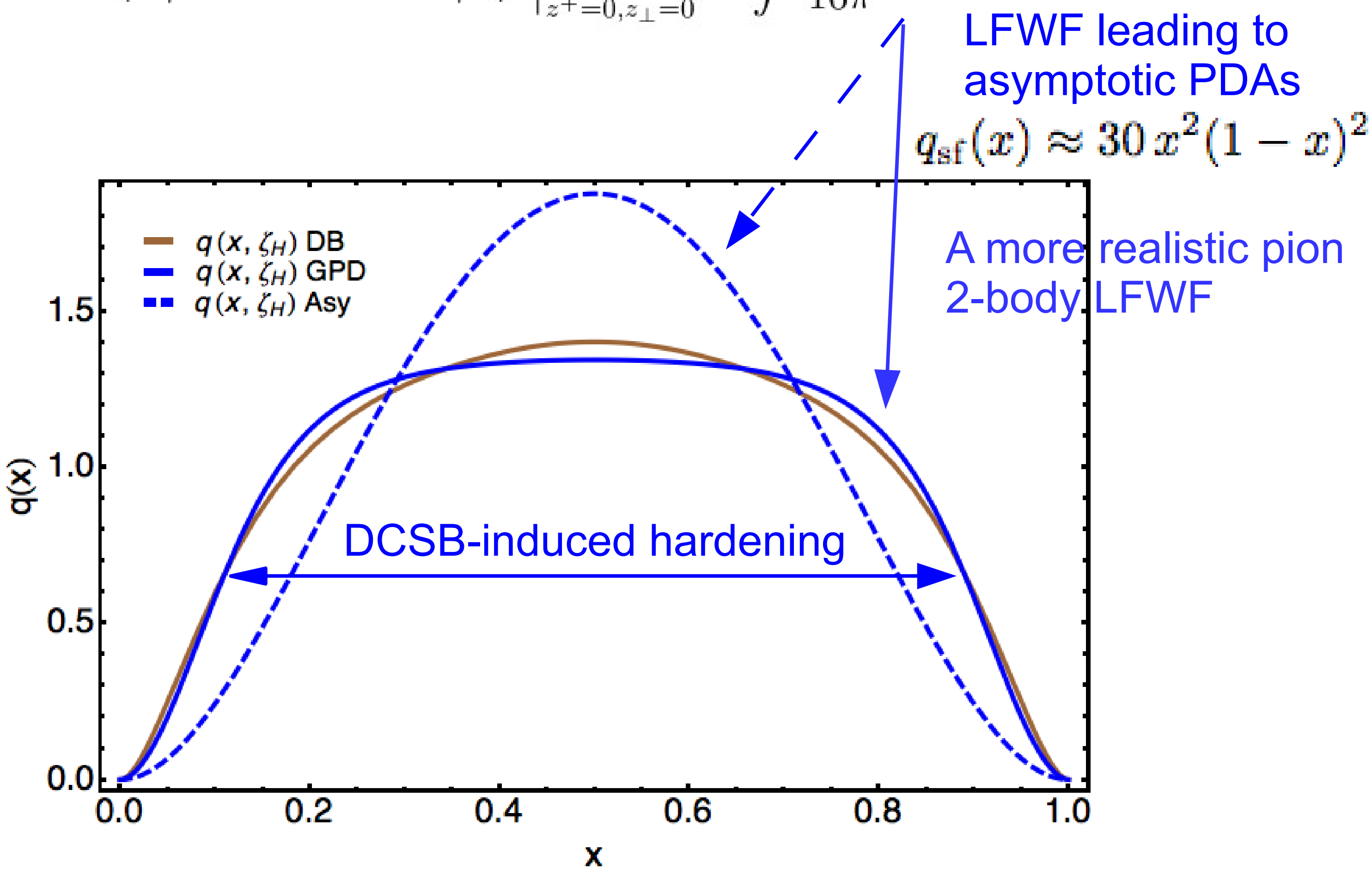

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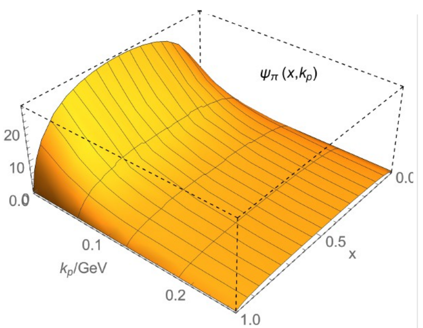

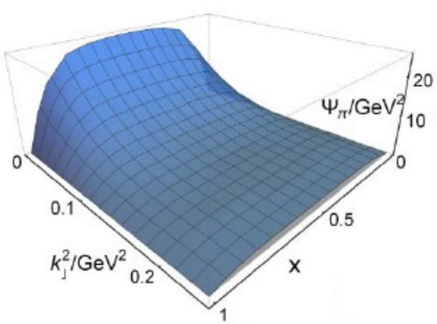

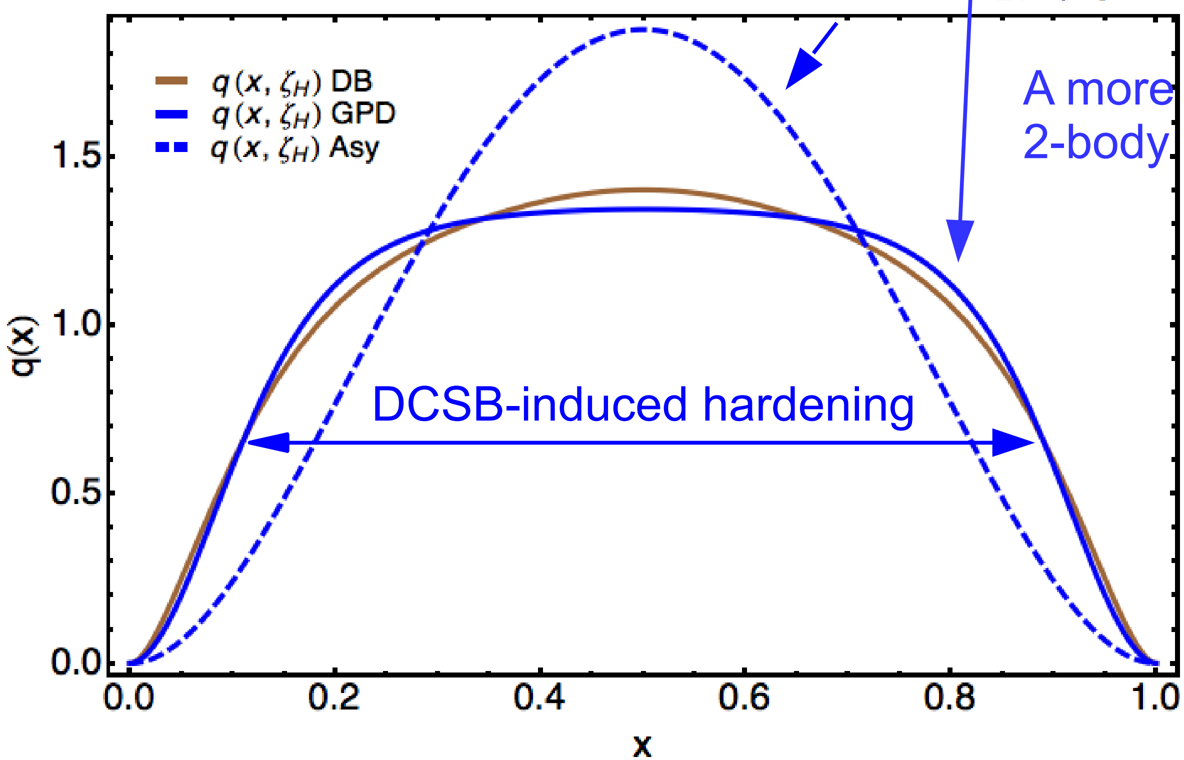

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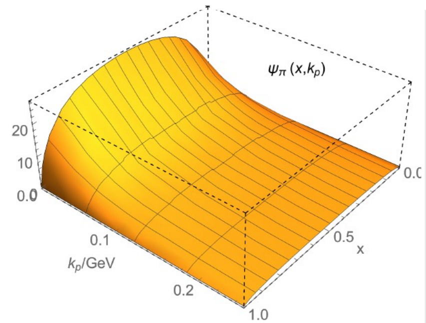

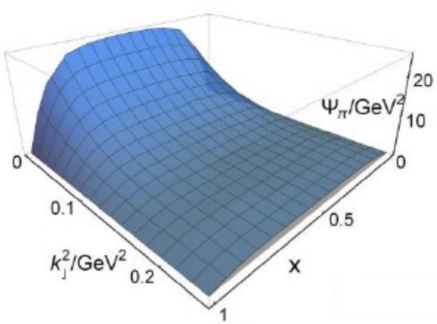

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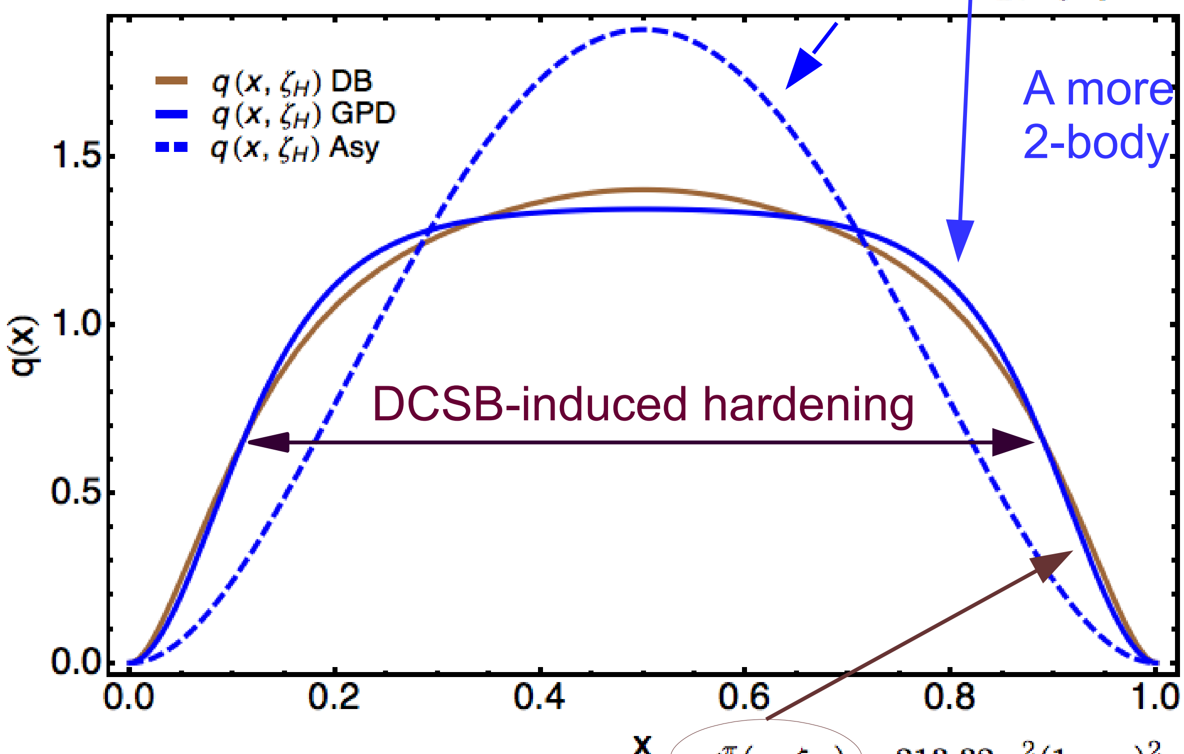

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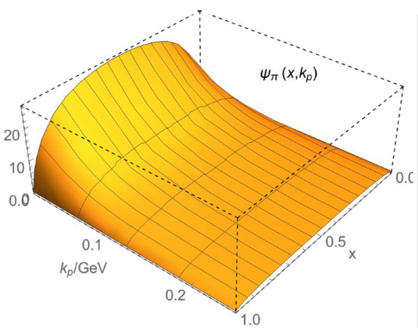

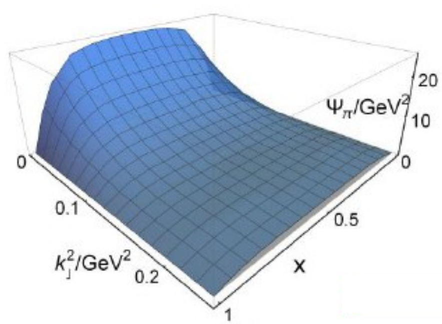

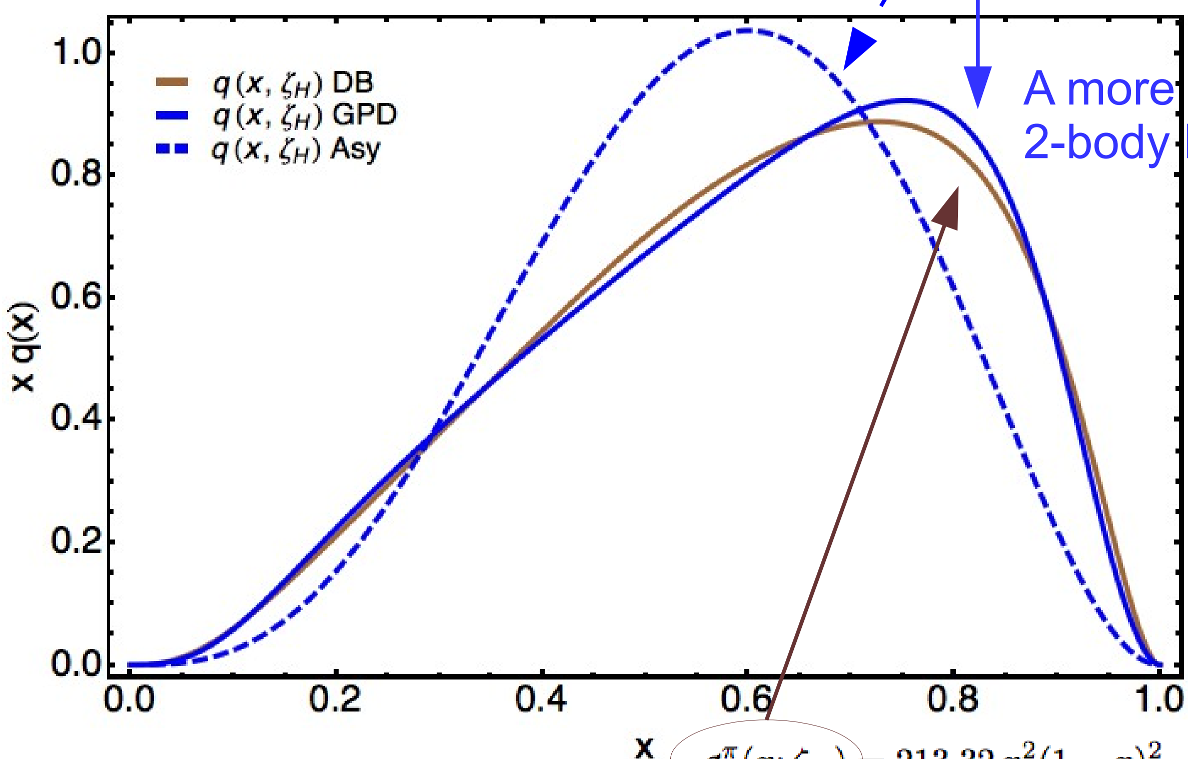

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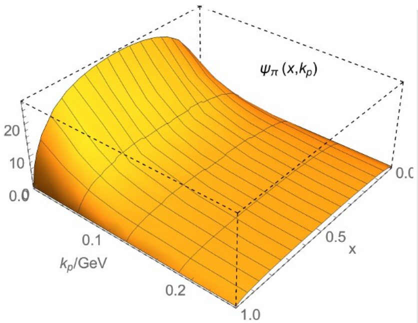

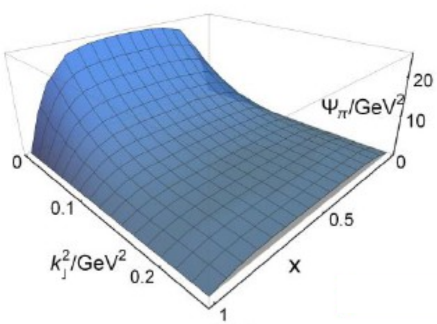

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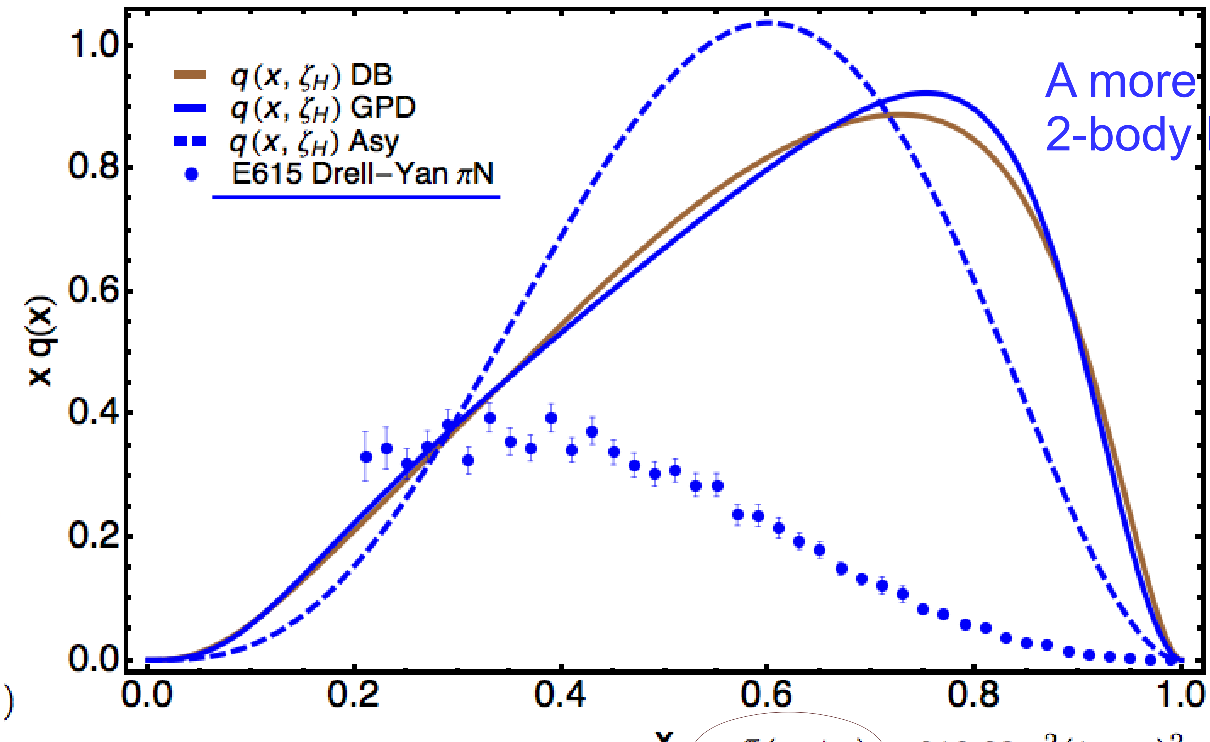

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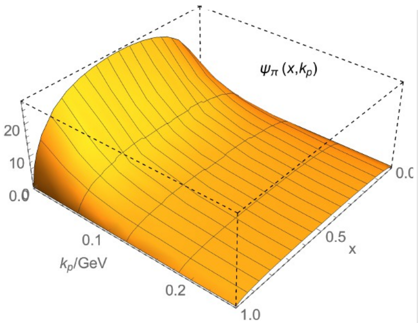

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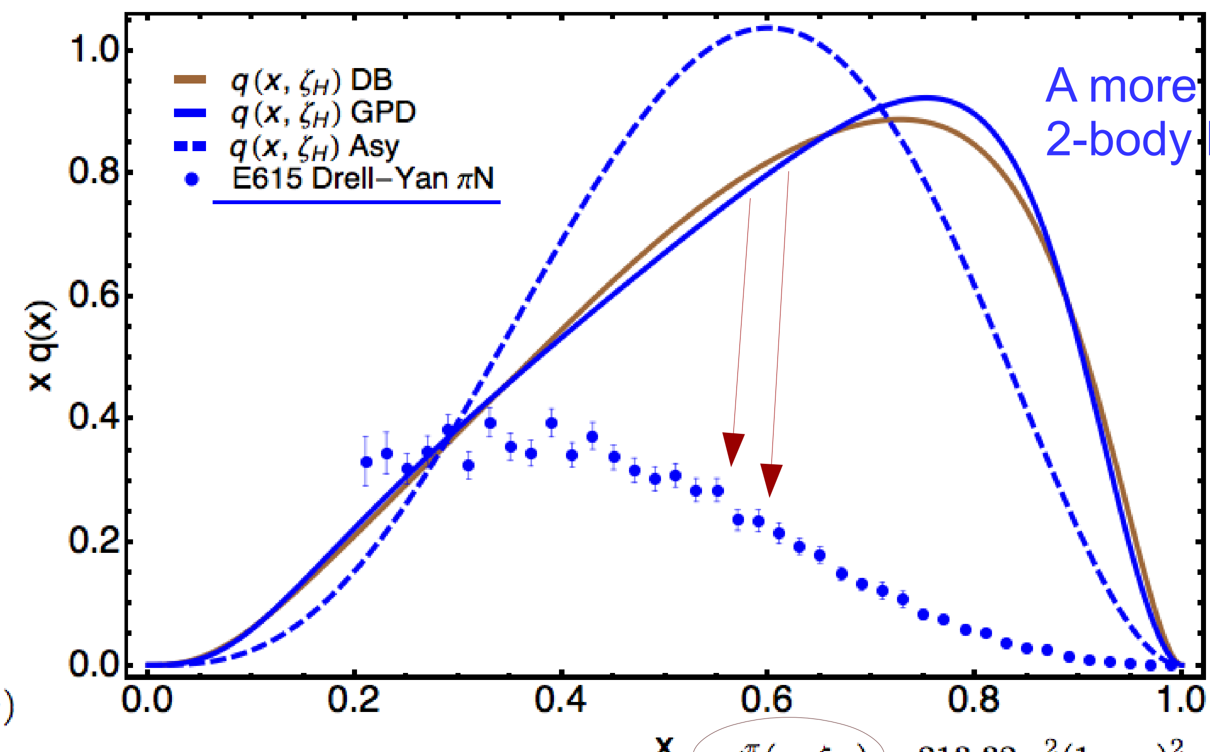

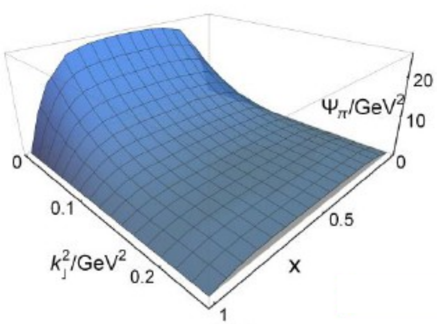

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$$M_n(t) = \int_0^1 dx\, x^n q(x,t)$$

$$t = \ln\left(\frac{\zeta^2}{\zeta_0^2}\right)$$

Moments' evolution (1-loop):

$$\frac{d}{dt} M_n(t) = -\frac{\alpha(t)}{4\pi} \gamma_0^n M_n(t) + \ldots$$

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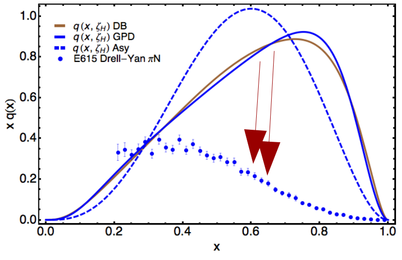

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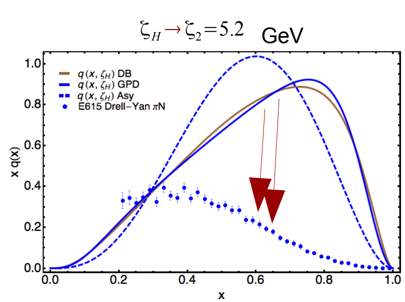

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$$P(x) = \frac{8}{3}\left(\frac{1+z^2}{(1-x)_+} + \frac{3}{2}\delta(x-1)\right)$$

$$\gamma_n = -\frac{4}{3}\left(3 + \frac{2}{(n+2)(n+3)} - 4\sum_{i=1}^{n+1}\frac{1}{i}\right)$$

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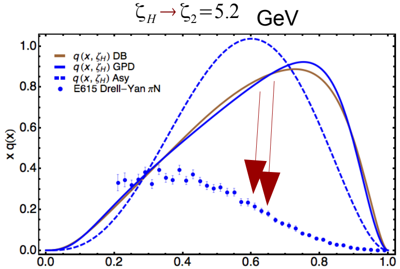

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$$\frac{d}{dt}\alpha(t) = -\frac{\alpha^2(t)}{4\pi}\beta_0 + \ldots$$

$$\gamma_0^n = -\frac{4}{3}\left(3 + \frac{2}{(n+2)(n+3)} - 4\sum_{i=1}^{n+1}\frac{1}{i}\right)$$

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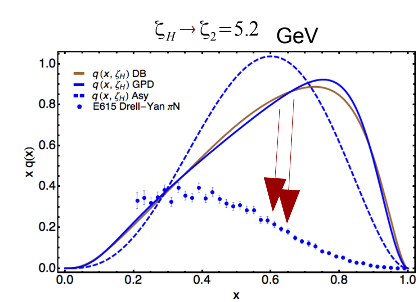

One application: pion PDF DGLAP evolution

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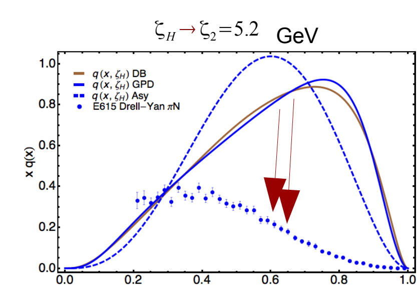

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Which value of Lambda?

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The evolution will thus depend on the scheme *via* the perturbative truncation

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The evolution will thus depend on the scheme *via* the perturbative truncation and the usual prejudice is that truncation errors are optimally small in MS scheme.

PDG2018:
[PRD98(2018)030001]

$$\Lambda^{(5)}_{\overline{MS}} = (210 \pm 14)\ \text{MeV}, \qquad (9.24b)$$

$$\Lambda^{(4)}_{\overline{MS}} = (292 \pm 16)\ \text{MeV}, \qquad (9.24c)$$

$$\Lambda^{(3)}_{\overline{MS}} = (332 \pm 17)\ \text{MeV}, \qquad (9.24d)$$

One application: pion PDF DGLAP evolution

Then, one can evolve the pion PDF, e.g. the one obtained by direct computation of Mellin moments, by using DGLAP evolution from one unknown hadronic scale up to the relevant one for the E615 experiment:

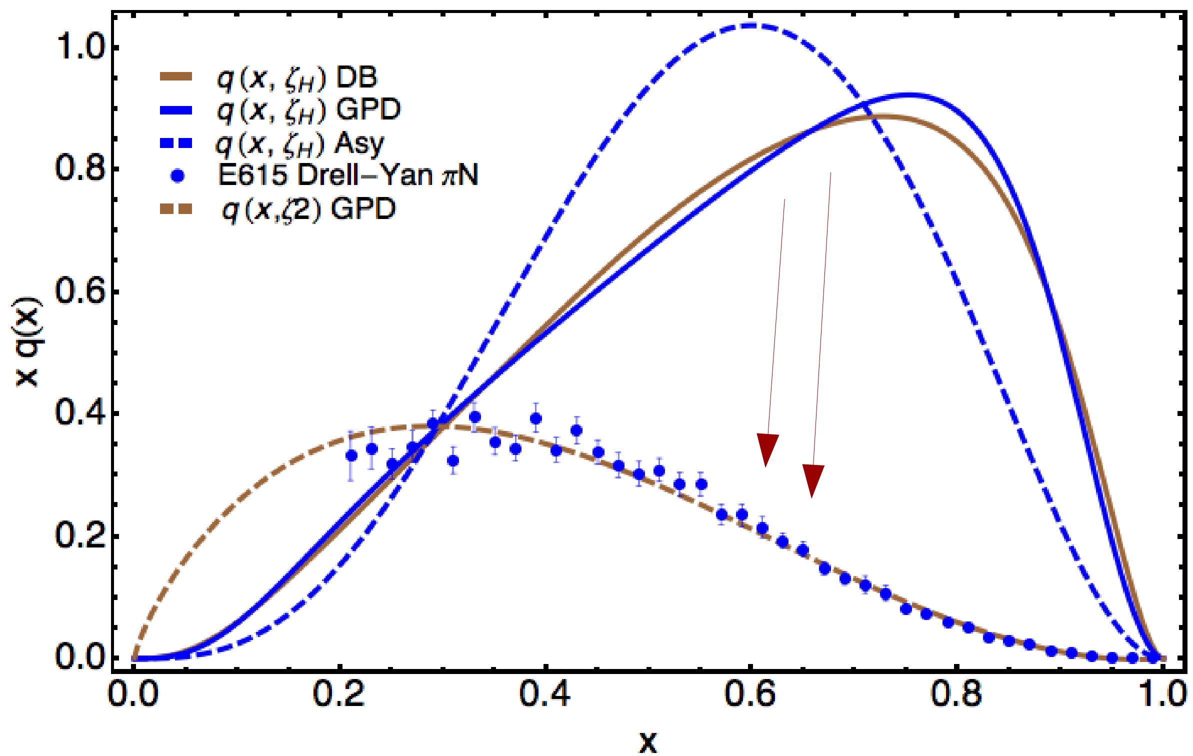
$$\langle x^m \rangle^{\pi}_{\zeta_H} = \int_0^1 dx\, x^m q^{\pi}(x;\zeta_H)$$
$$= \frac{N_c}{n\cdot P} \mathrm{tr} \int_{dk} \left[\frac{n\cdot k_\eta}{n\cdot P}\right]^m \Gamma_\pi(k_{\bar\eta},P)\, S(k_{\bar\eta})\, n\cdot\partial_{k_\eta} \left[\Gamma_\pi(k_\eta,-P) S(k_\eta)\right]$$

$$q^{\pi}(x;\zeta_H) = 213.32\, x^2 (1-x)^2 \times \left[1 - 2.9342\sqrt{x(1-x)} + 2.2911\, x(1-x)\right]$$

$$\zeta_H \rightarrow \zeta_2 = 5.2 \text{ GeV}$$

Optimal best-fitting parameters:

$$\Lambda_{QCD} = 0.234 \text{ GeV} \; ; \quad \zeta_H = 0.349 \text{ GeV} \; .$$



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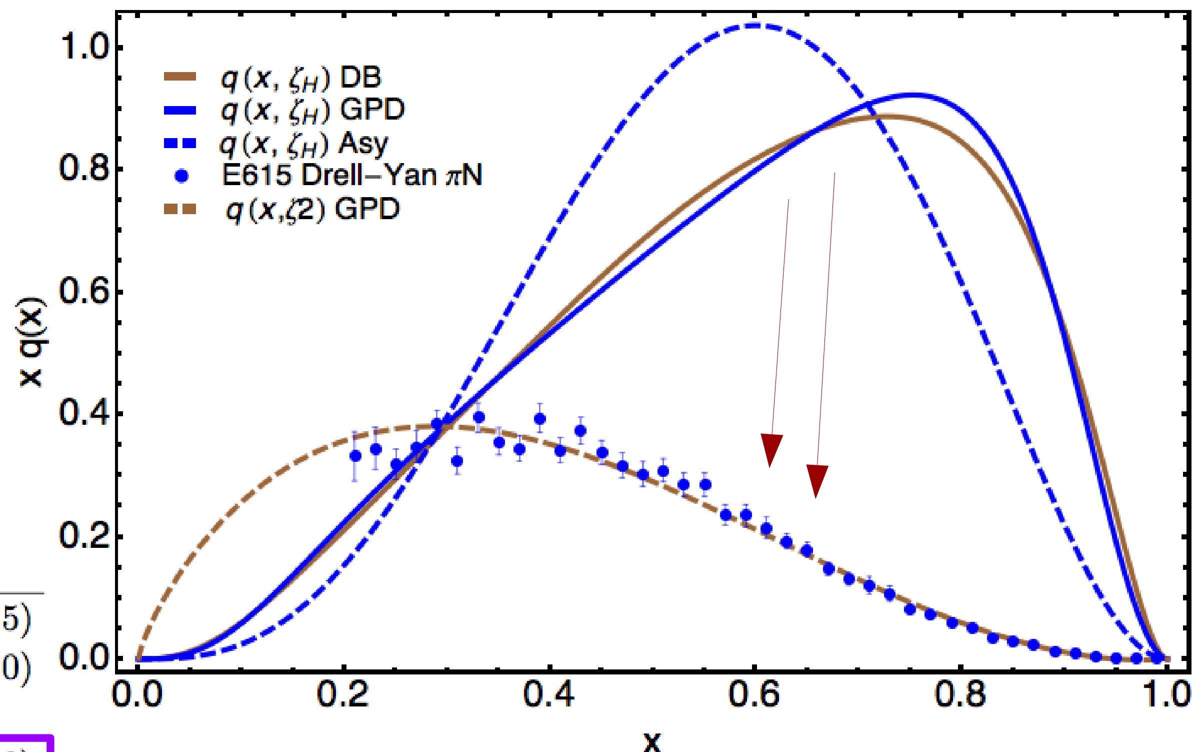

ζ_2	$\langle x\rangle^\pi_u$	$\langle x^2\rangle^\pi_u$	$\langle x^3\rangle^\pi_u$
Ref. [33]	0.24(2)	0.09(3)	0.053(15)
Ref. [34]	0.27(1)	0.13(1)	0.074(10)
Ref. [35]	0.21(1)	0.16(3)	
average	0.24(2)	0.13(4)	0.064(18)
Herein	0.24(2)	0.098(10)	0.049(07)

Comparison with the three first moments obtained from lQCD

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$$\zeta_H \rightarrow \zeta_L = 2 \text{ GeV} \rightarrow \zeta_2 = 5.2 \text{ GeV}$$

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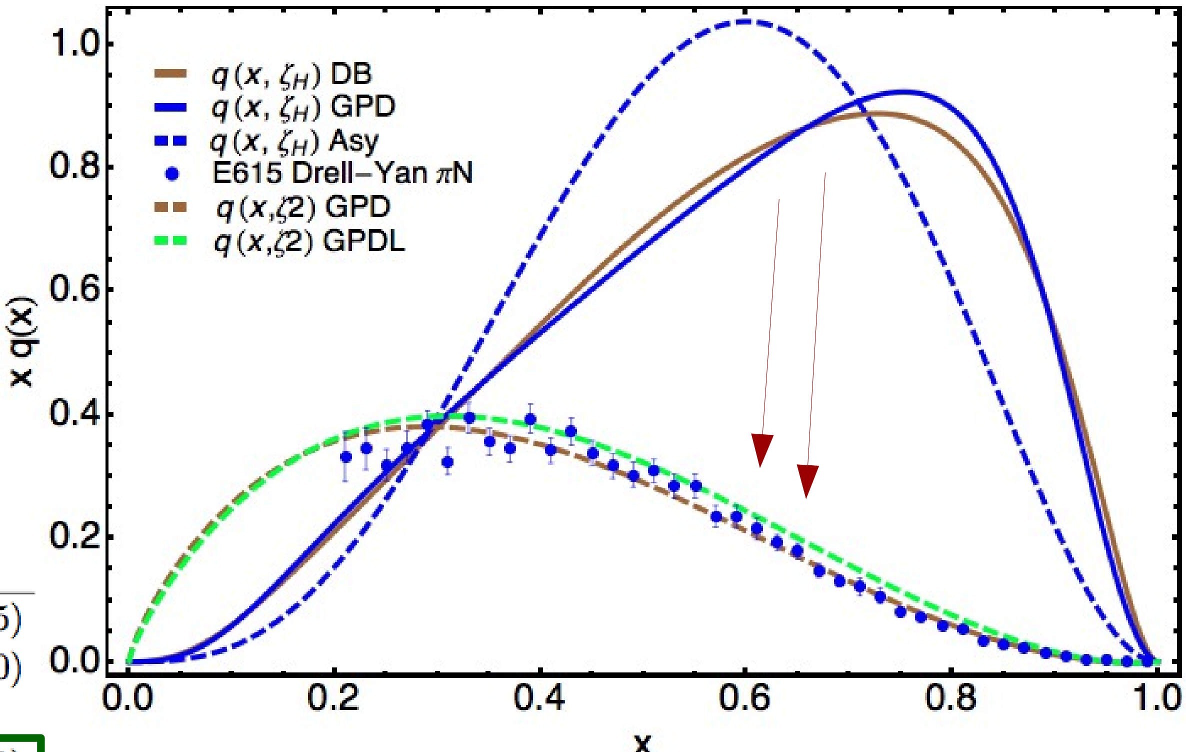

Matching the three first moments obtained from lQCD

One application: pion PDF DGLAP evolution

Which value of Lambda? It depends on the scheme... Indeed, at the one-loop level, its value defines by itself the scheme!!!

$$\alpha(t) = \frac{4\pi}{\beta_0 (t - t_\Lambda)} + \ldots = \frac{4\pi}{\beta_0 \ln\left(\frac{\zeta^2}{\Lambda^2}\right)} + \ldots$$

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The evolution will thus depend on the scheme *via* the perturbative truncation

The use of $\Lambda = 0.234$ GeV can be thus interpreted as the choice of particular scheme, differing from MS.

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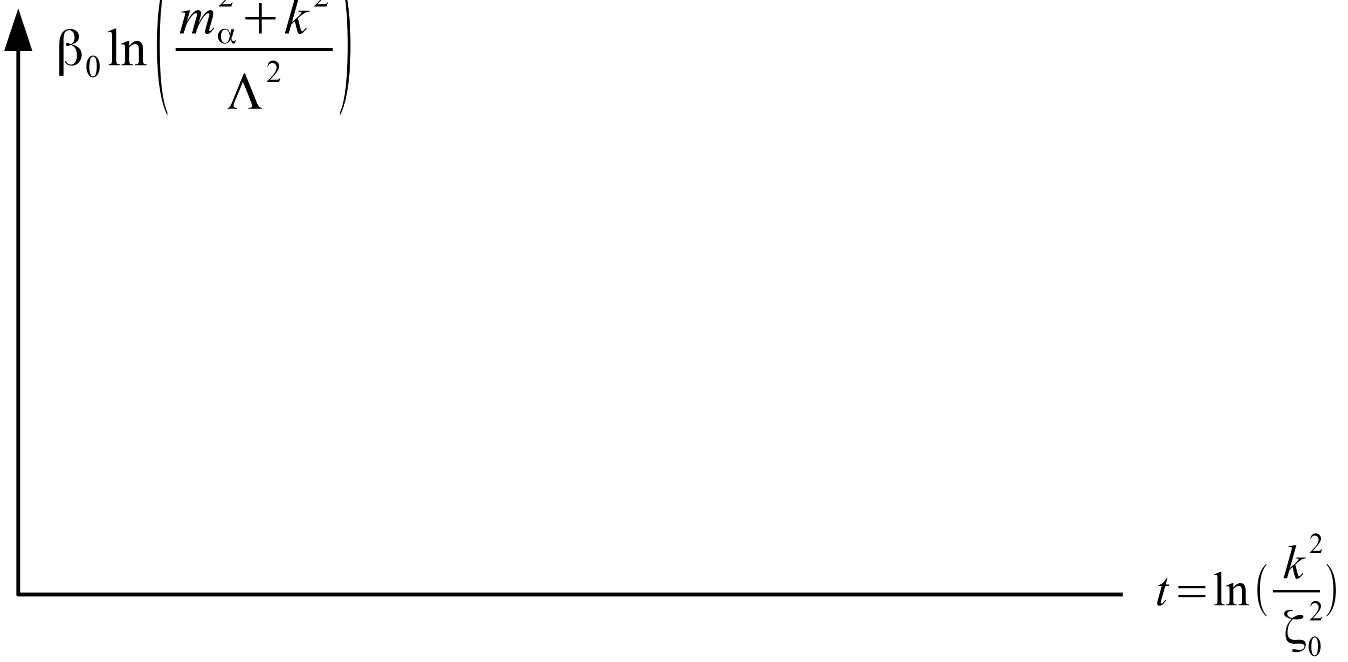
The evolution will thus depend on the scheme *via* the perturbative truncation

The use of $\Lambda = 0.234$GeV can be thus interpreted as the choice of particular scheme, differing from MS. Beyond this, the scheme can be defined in such a way that one-loop DGLAP is exact at all orders (Grunberg's effective charge).

One application: pion PDF DGLAP evolution

$$\alpha(t) = \frac{4\pi}{\beta_0 \ln\left(\frac{m_\alpha^2 + \zeta_0^2 \exp(t)}{\Lambda^2}\right)} = \frac{4\pi}{\beta_0 \ln\left(\frac{m_\alpha^2 + k^2}{\Lambda^2}\right)}$$



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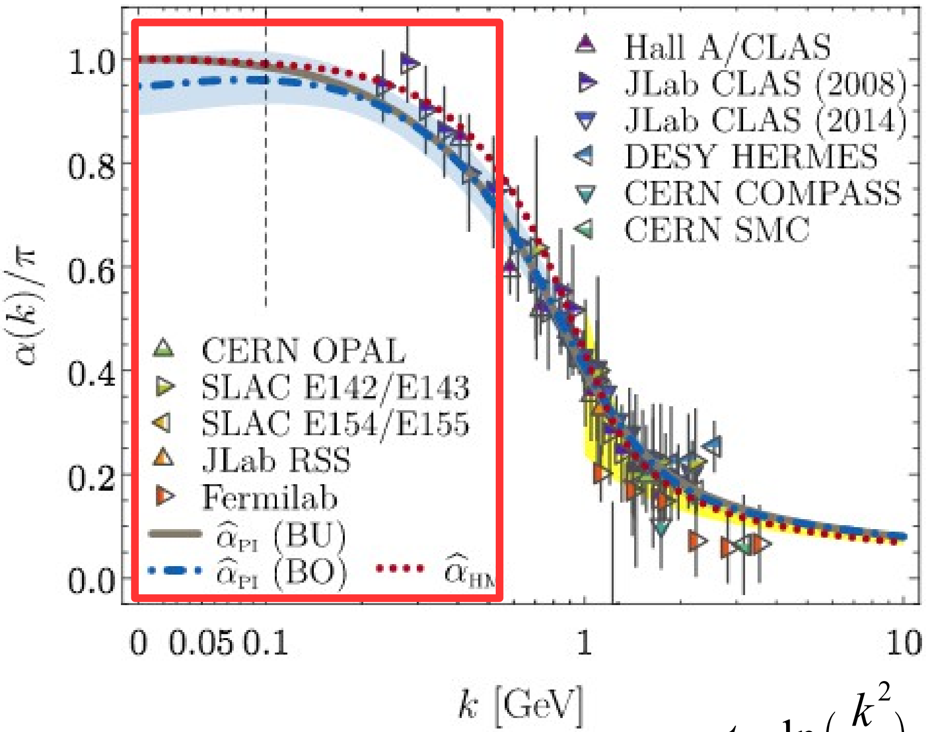
$$\alpha(0) = \alpha_{PI}(0) \;\rightarrow\; m_\alpha = 0.300 \text{ GeV}$$



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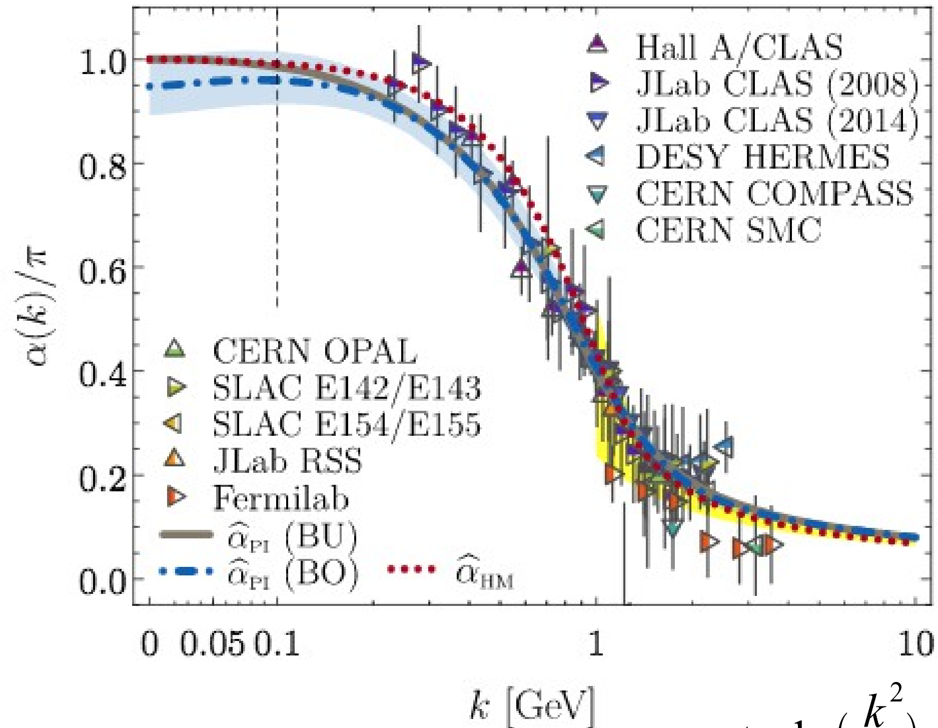
$$\frac{d}{dt}M_n(t) = -\frac{\alpha(t)}{4\pi}\gamma_0^n M_n(t)$$

Numerical integration with the effective charge

$$M_n(t) = M_n(t_0)\exp\left(-\frac{\gamma_0^n}{4\pi}\int_{t_0}^{t} dz\,\alpha(z)\right)$$



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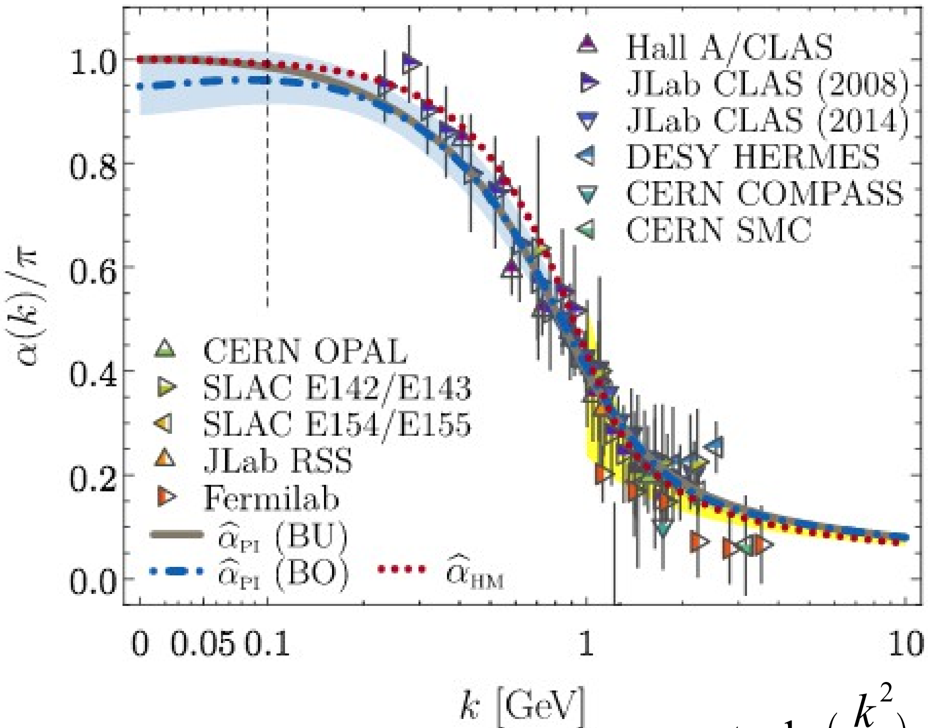

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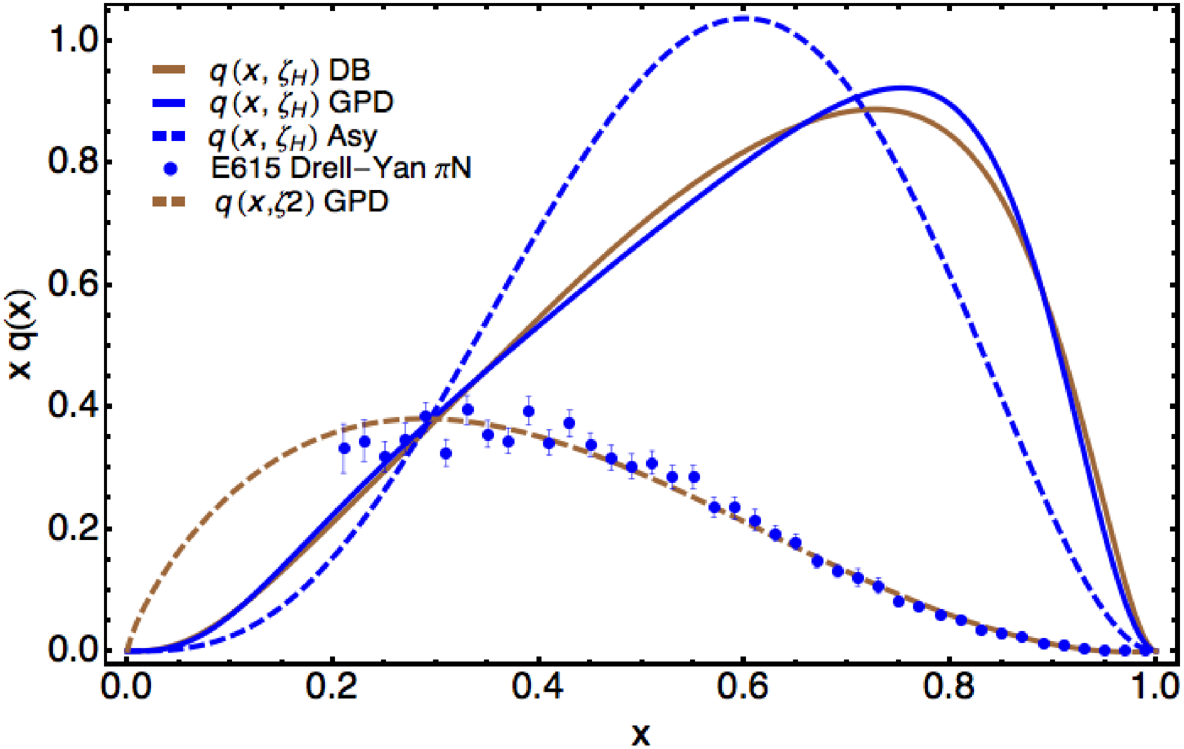
If one identifies: $m_\alpha \equiv \zeta_H$, all the scales (and the evolution between them) appear thus fixed, apart from Λ_{QCD} (fixed by the scheme).

One application: pion PDF DGLAP evolution

Then, one can evolve the pion PDF, e.g. the one obtained by direct computation of Mellin moments, by using DGLAP evolution from one unknown hadronic scale up to the relevant one for the E615 experiment:

$$\Lambda_{QCD} = 0.234 \text{ GeV}; \; \zeta_H \equiv m_\alpha \rightarrow \zeta_2 = 5.2 \text{ GeV}$$



If one identifies: $m_\alpha \equiv \zeta_H$, all the scales (and the evolution between them) appear thus fixed, apart from Λ_{QCD} (fixed by the scheme). And the agreement with E615 data is perfect!!!

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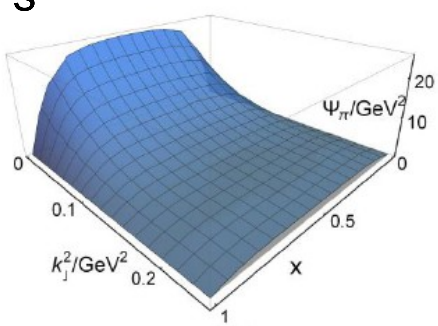
Then, one can evolve the pion PDF, e.g. the one obtained by direct computation of Mellin moments, by using DGLAP evolution from one unknown hadronic scale up to the relevant one for the E615 experiment:

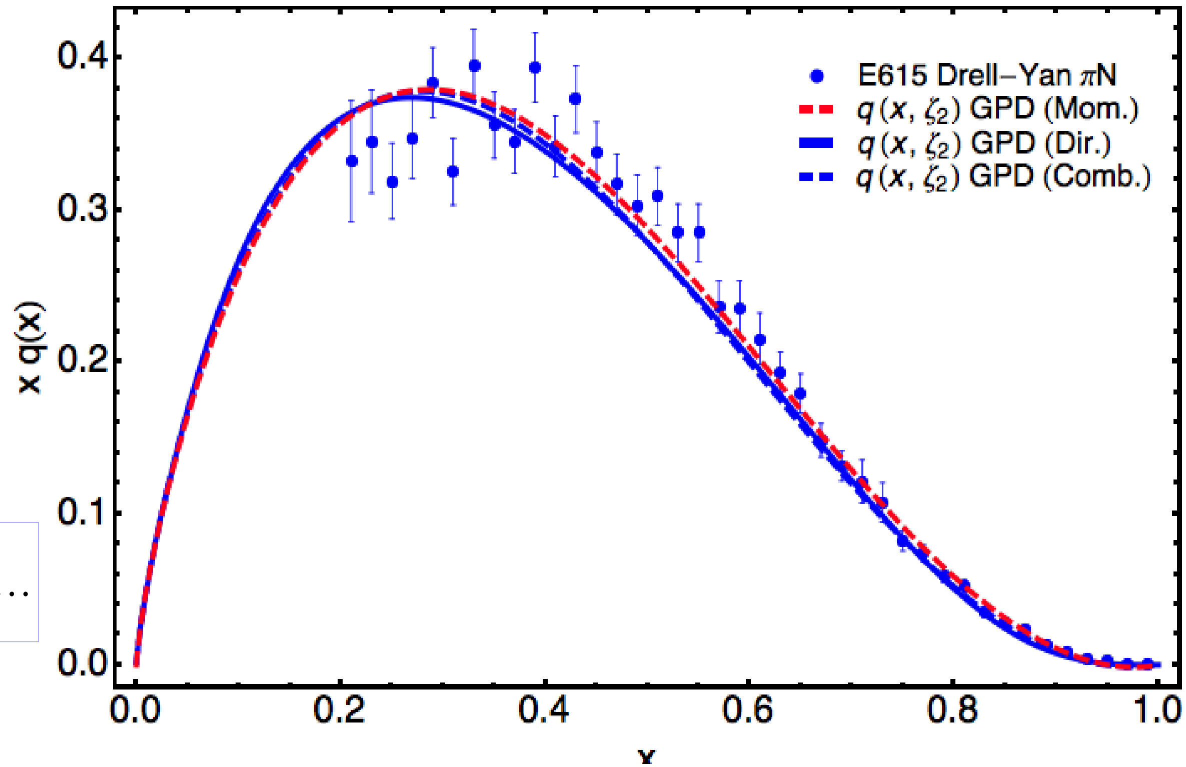
The same is obtained from the overlap of realistic pion 2-body LFWFs



and after integration of the DGLAP master equation

$$\frac{d}{dt}q(x,t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y} q(y,t) P\left(\frac{x}{y}\right) + \dots$$

$\Lambda_{QCD} = 0.234$ GeV; $\zeta_H \equiv m_\alpha \rightarrow \zeta_2 = 5.2$ GeV



If one identifies: $m_\alpha \equiv \zeta_H$, all the scales (and the evolution between them) appear thus fixed, apart from Λ_{QCD} (fixed by the scheme). And the agreement with E615 data is perfect!!!

Conclusions

- One can define a parameter-free effective charge (completely determined from 2-points gauge sector), with no Landau pole (physical coupling showing an IR fixed point) and smoothly connecting IR and UV domains (no explicit matching procedure)

- It is shown to be in good agreement with the Bjorken-rule effective charge and with the coupling from the light-front holographic model.

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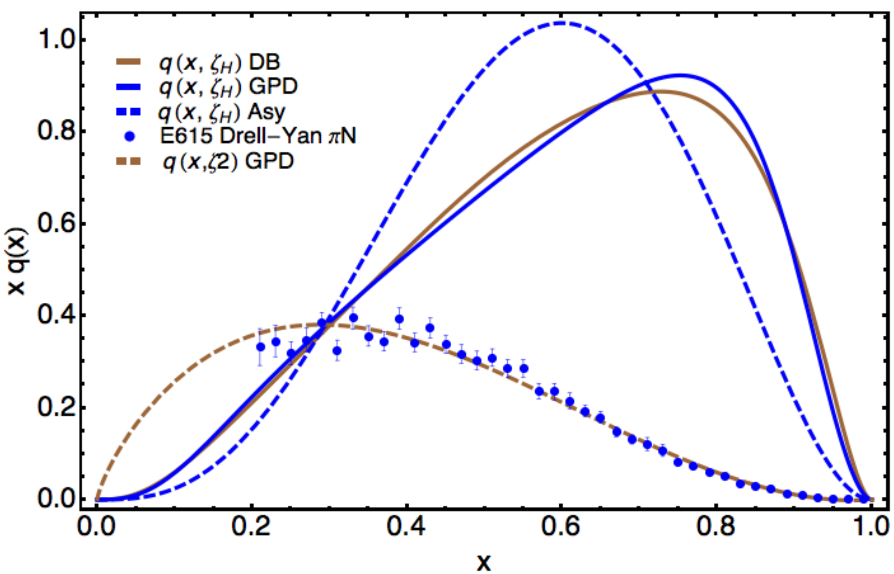

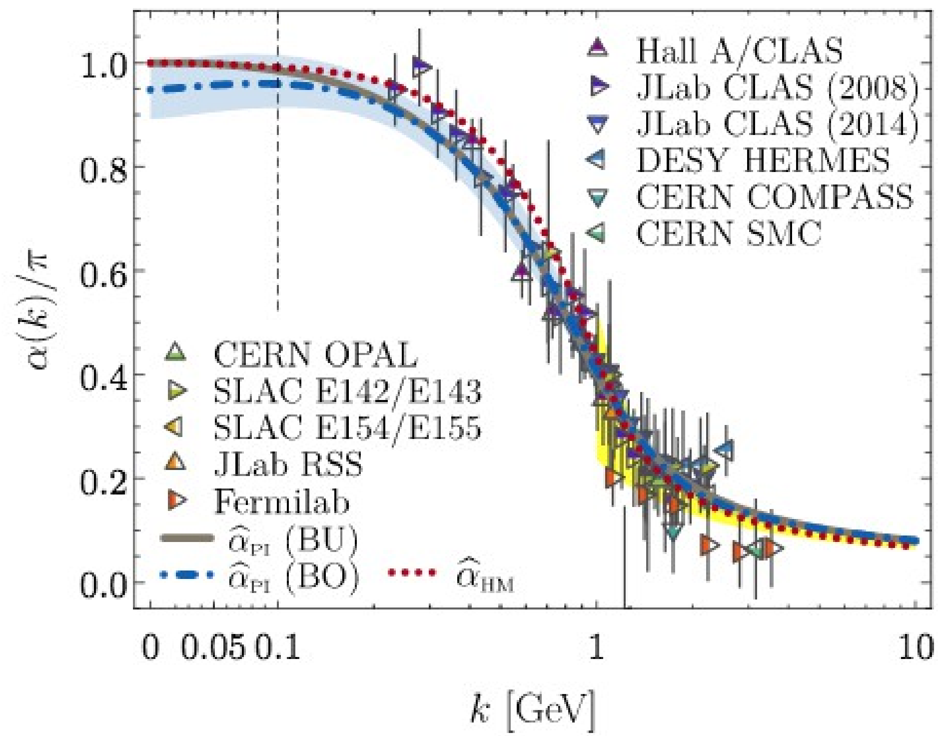

- The PI effective charge is shown to be equivalent, within the IR domain, to the one which can be defined to get an "exact" (at all order in perturbations) DGLAP evolution for the pion PDF.