# Nuclear models for neutrino-nucleus interactions in long baseline neutrino oscillation experiments


**Maria B. Barbaro**

**University of Turin, Italy**
**and**
**INFN - Turin**

Istituto Nazionale di Fisica Nucleare
Sezione di Torino



IOP Institute of Physics

INPC 2019 Glasgow, UK

**International Nuclear Physics Conference 2019**

**29 July – 2 August 2019, Scottish Event Campus, Glasgow, UK**


# Motivations: Neutrino Physics

## 1. Interpretation of long baseline neutrino experiments

- measurement of **CP violation** in the leptonic mixing matrix
  - improved accuracy for **oscillation angles**
  - neutrino **mass ordering** (NH or IH)

through appearance and disappearance of neutrinos of given flavour.
Detectors are made of **complex nuclei** (C, O, Ar,…) and need reliable nuclear models for data analyses.

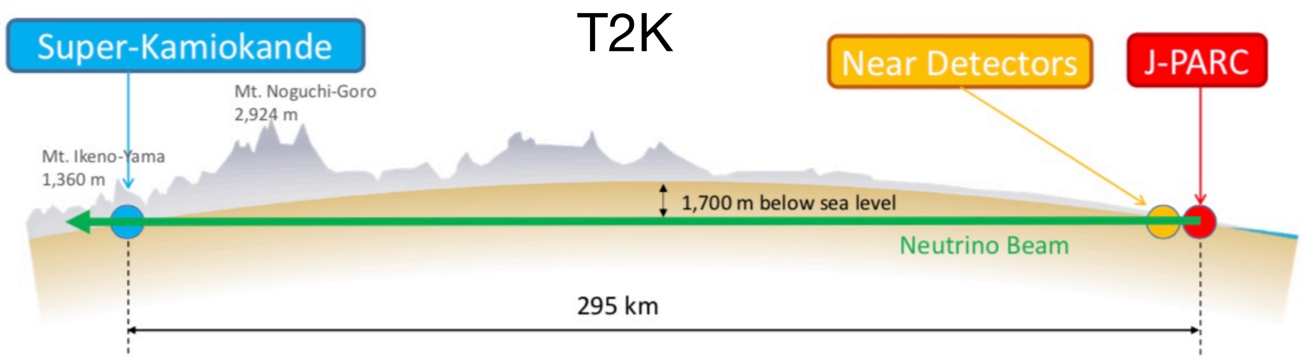
Essential total systematic uncertainty <3% for DUNE/T2HK

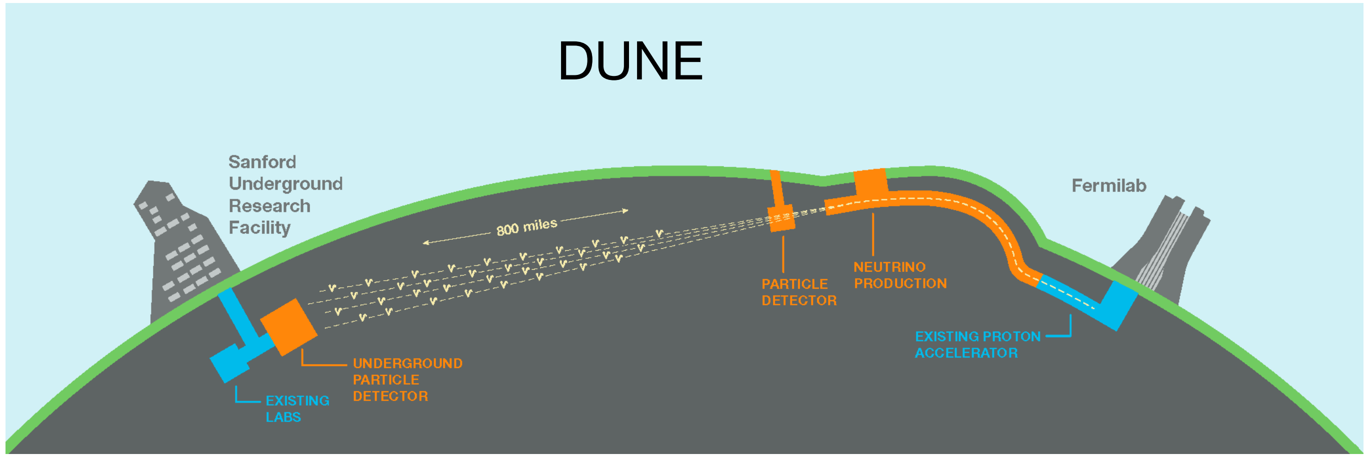
**Large systematic uncertainty comes from modelling of neutrino-nucleus interactions.**

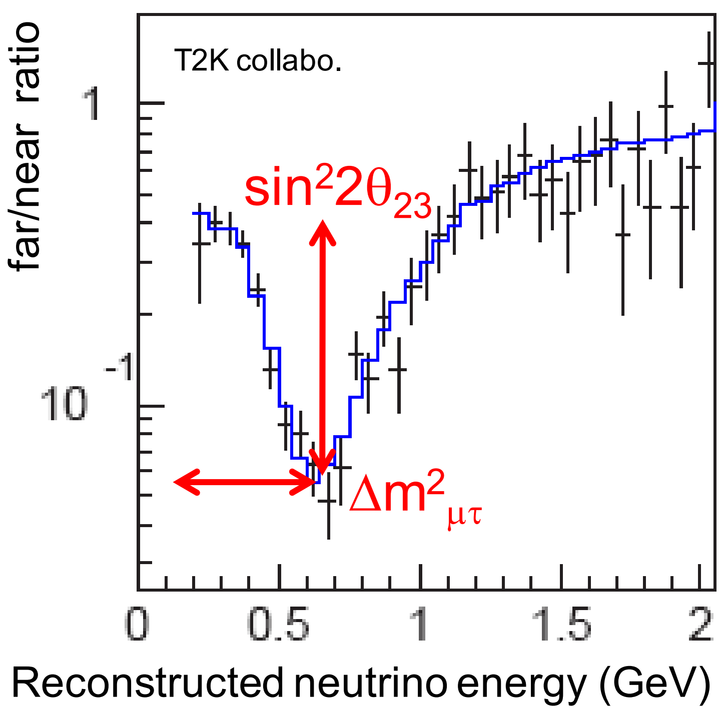
$$P_{\alpha\to\beta} = \left|\langle\nu_\alpha|\nu_\beta(t)\rangle\right|^2 = \left|\sum_i U^*_{\alpha_i} U_{\beta_i} e^{im_i^2 L/2E_\nu}\right|^2$$

Neutrino energy —> mass-squared splitting (position)
mixing angle (amplitude)

**Problem**: $E_\nu$ is not known. It must be reconstructed using a **nuclear model**.

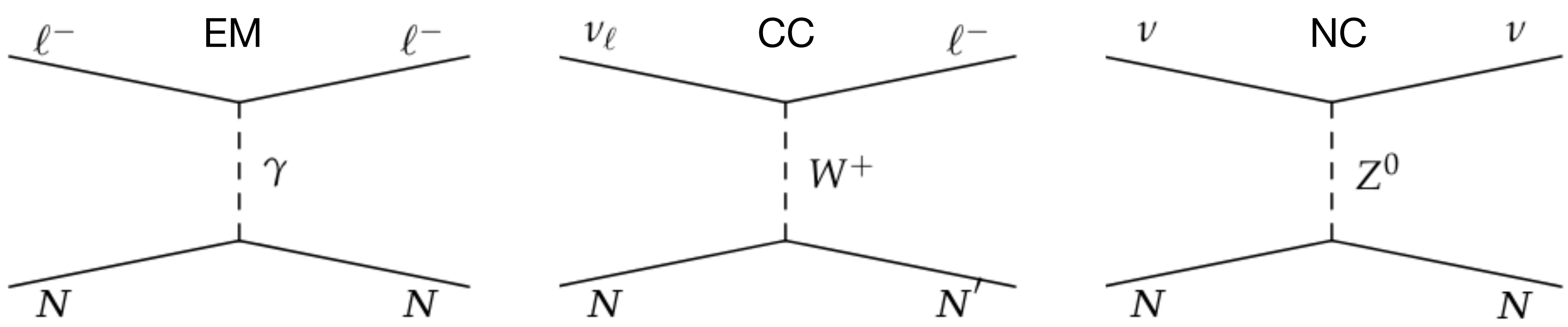


## 2. Neutrinos as a probe of nuclear structure and dynamics

- neutrinos can provide useful information on the nucleus, complementary to what can be known from charged lepton- and photon-nucleus scattering




(a) Electromagnetic scattering (b) Charged-current scattering (c) Neutral-current scattering

- From the **theoretical** point of view the $e - A$ and $\nu - A$ processes are strictly related:

- the vector EM and weak currents are connected by CVC
- however, neutrinos probe both the **vector** and the **axial** currents: richer structure of the cross section

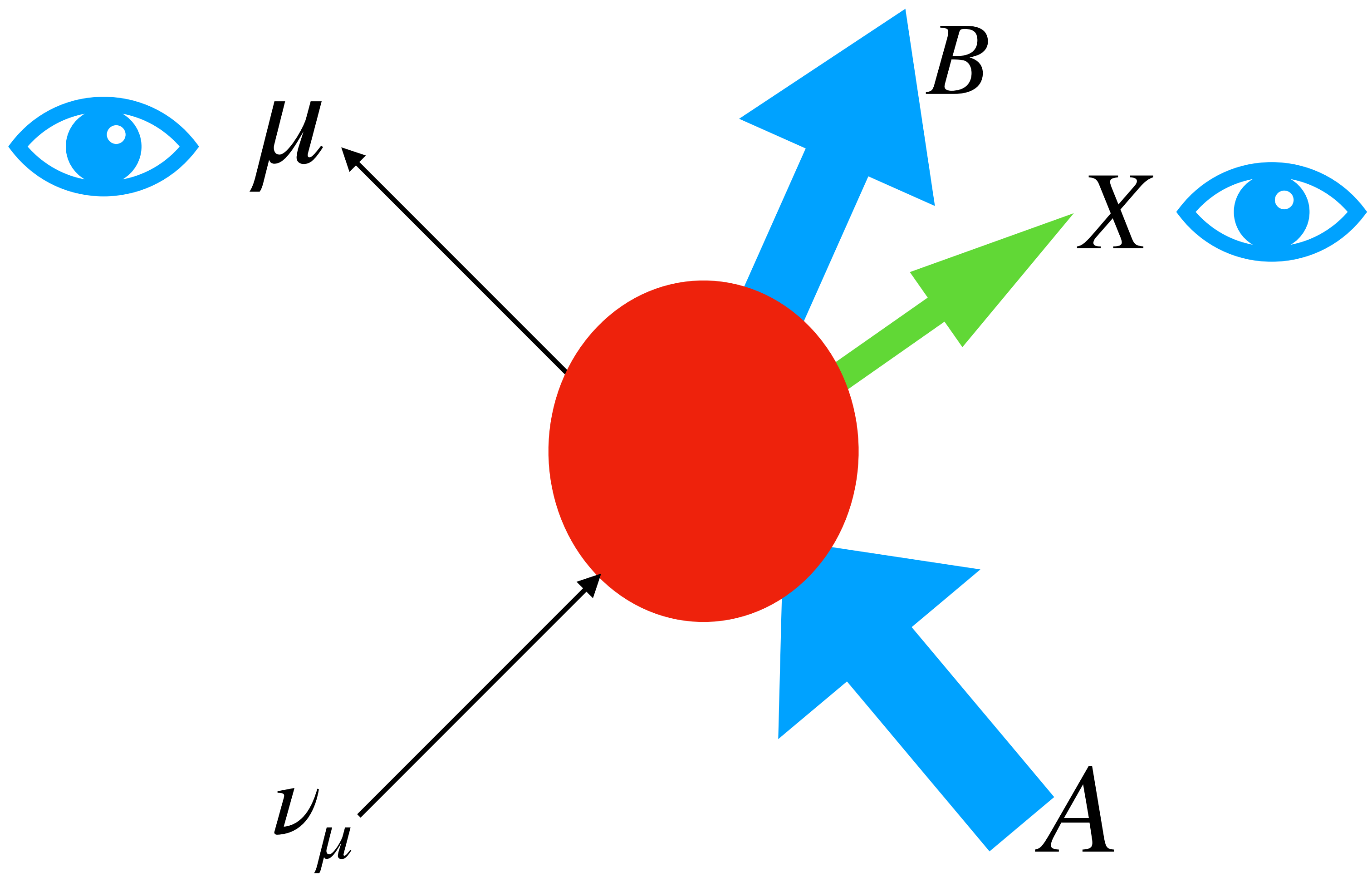
A good nuclear model must be able to describe **simultaneously** the two processes.
**Validation against electron scattering data** is a necessary (albeit not sufficient) test and a valid benchmark for nuclear models to be used in neutrino studies.

**Inclusive:** $(\nu_\mu, \mu)$

only the outgoing lepton is observed

**Semi-inclusive:** $(\nu_\mu, \mu X)$

$\mu$ and $X$ detected in coincidence




A good nuclear model should also be able to describe both inclusive and semi-inclusive processes. The latter are far more sensitive to the detailed treatment of nuclear structure and dynamics.

# Formalism

Double differential neutrino (+) or antineutrino (-) Charged Current cross section on a nucleus
**Inclusive** case: only the final lepton is detected, $e.g.\ (\nu_\mu, \mu)$

$$\left[\frac{d\sigma}{dk_\mu d\Omega_\mu}\right]_\pm \sim \eta_{\mu\nu} W^{\mu\nu} = \sigma_0 \left(V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_T R_T \pm V_{T'}R_{T'}\right)$$

Rosenbluth-like separation:
**5 response functions**

$\hat{z} = \hat{q}$

$\eta_{\mu\nu}$ leptonic tensor

$$\sigma_0 = \frac{G_F^4 \cos^2\theta_C}{2\pi^2}\left(k_\mu \cos\frac{\tilde{\theta}_\mu}{2}\right)^2$$

$$W^{\mu\nu} = \overline{\sum} \delta(E_f - E_i - \omega) < f\,|\,J^\mu)Q)\,|\,i >^* < f\,|\,J^\nu(Q)\,|\,i > \text{ hadronic tensor}$$

$J^\mu$ nuclear current, $|\,i, f >$ nuclear states

Each response has **vector and axial components**, from V and A leptonic and hadronic currents and depends on **2 variables**

$$R_K = R_K^{VV} + R_K^{AA}\ , \qquad K = CC, CL, LL, T$$

$$R_{T'} = R_{T'}^{VA}$$

$$R_K \equiv R_K(q, \omega)$$

Comparison with **electron scattering** (e,e')

$$\frac{d\sigma}{dk_e d\Omega_e} = \sigma_{Mott}\left(v_L R_L^{VV} + v_T R_T^{VV}\right)$$

only 2 vector responses

**Semi-inclusive** case $(\nu_l, lN)$:

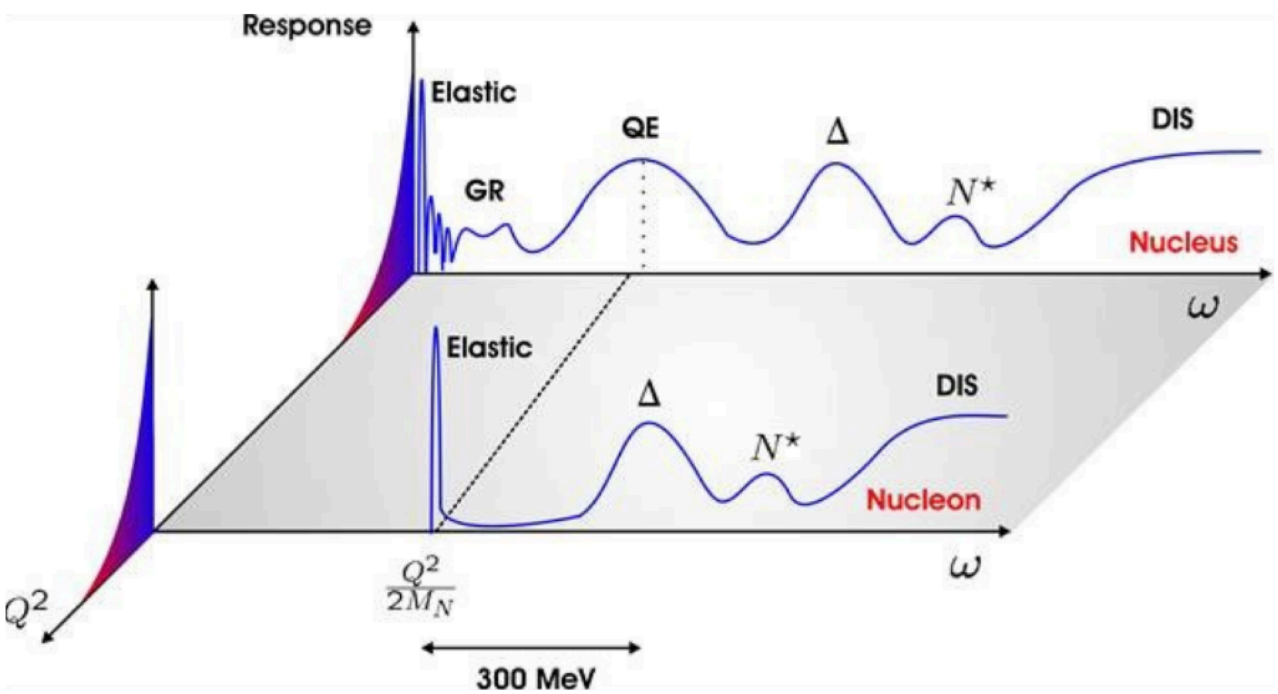
**10 responses, 5 variables**

# Electron versus neutrino-nucleus scattering

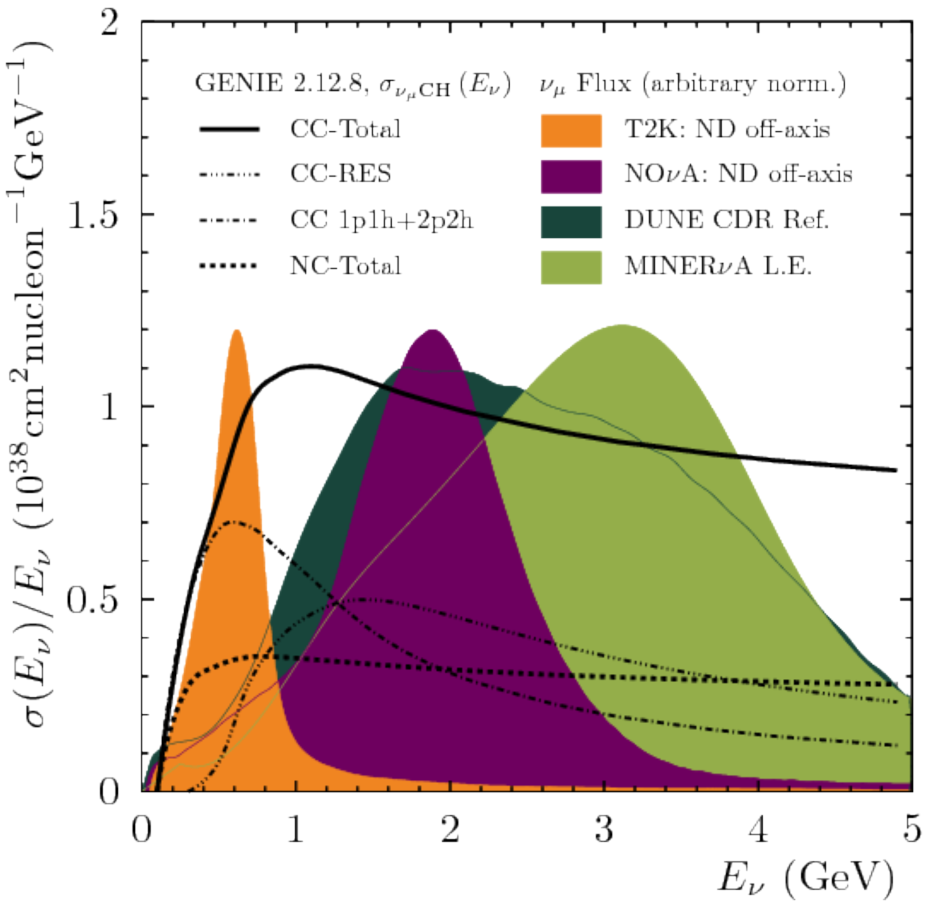
- The **experimental** situation for electron and neutrino scattering is different

- $(e, e')$: the incoming electron energy is well determined and different channels can be clearly identified by knowing the energy and momentum transfer

- $(\nu_l, l)$: the neutrino energy is broadly distributed in the neutrino beam and different channels and different nuclear effects can contribute to the same kinematics of the outgoing lepton




K. Mahn et al., Ann.Rev.Nucl.Part.Sci. 68 (2018)



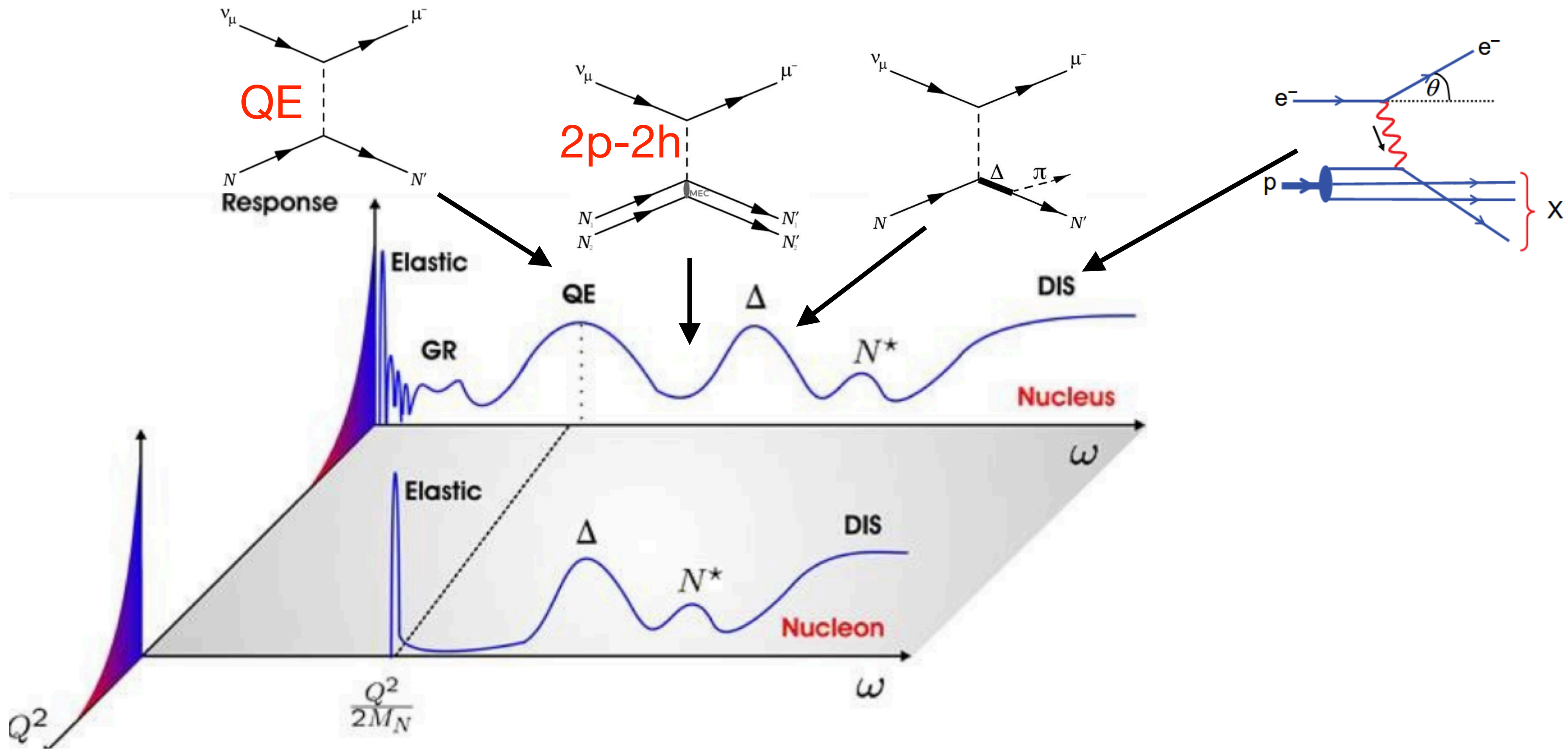

**$E_\nu \sim 1$ GeV: relativistic nuclear models are needed**

# Nuclear response to electroweak probes

## Neutrino beams are not monochromatic: multi-scale problem

When integrating over the neutrino flux, different processes contribute to the cross section, depending on the energy transferred from the neutrino to the nucleus

$$\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu} = \frac{1}{\Phi_{tot}} \int \left[\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu}\right]_{E_\nu} \Phi(E_\nu) dE_\nu$$




- Quasi-elastic scattering (CCQE): $\nu_l n \to l^- p, \qquad \bar{\nu}_l p \to l^+ n$
- Two-nucleon knockout: $\nu_l NN \to l^- NN, \qquad \bar{\nu}_l NN \to l^+ NN$
- Resonance production: $\nu_l p \to l^- \Delta^{++}, \quad \Delta^{++} \to p\pi^+$ and $\bar{\nu}_l n \to l^+ \Delta^-, \quad \Delta^- \to n\pi^-$
- Deep inelastic scattering $\nu_l/\bar{\nu}_l(k) + N(p) \to \mu^{\mp}(k') + X(p')$
- Coherent meson production $\nu_l + A \to l^- + m^+ + A, \quad \bar{\nu}_l + A \to l^+ + m^- + A$ with $m^\pm = \pi^\pm, K^\pm, \rho^\pm, \ldots$

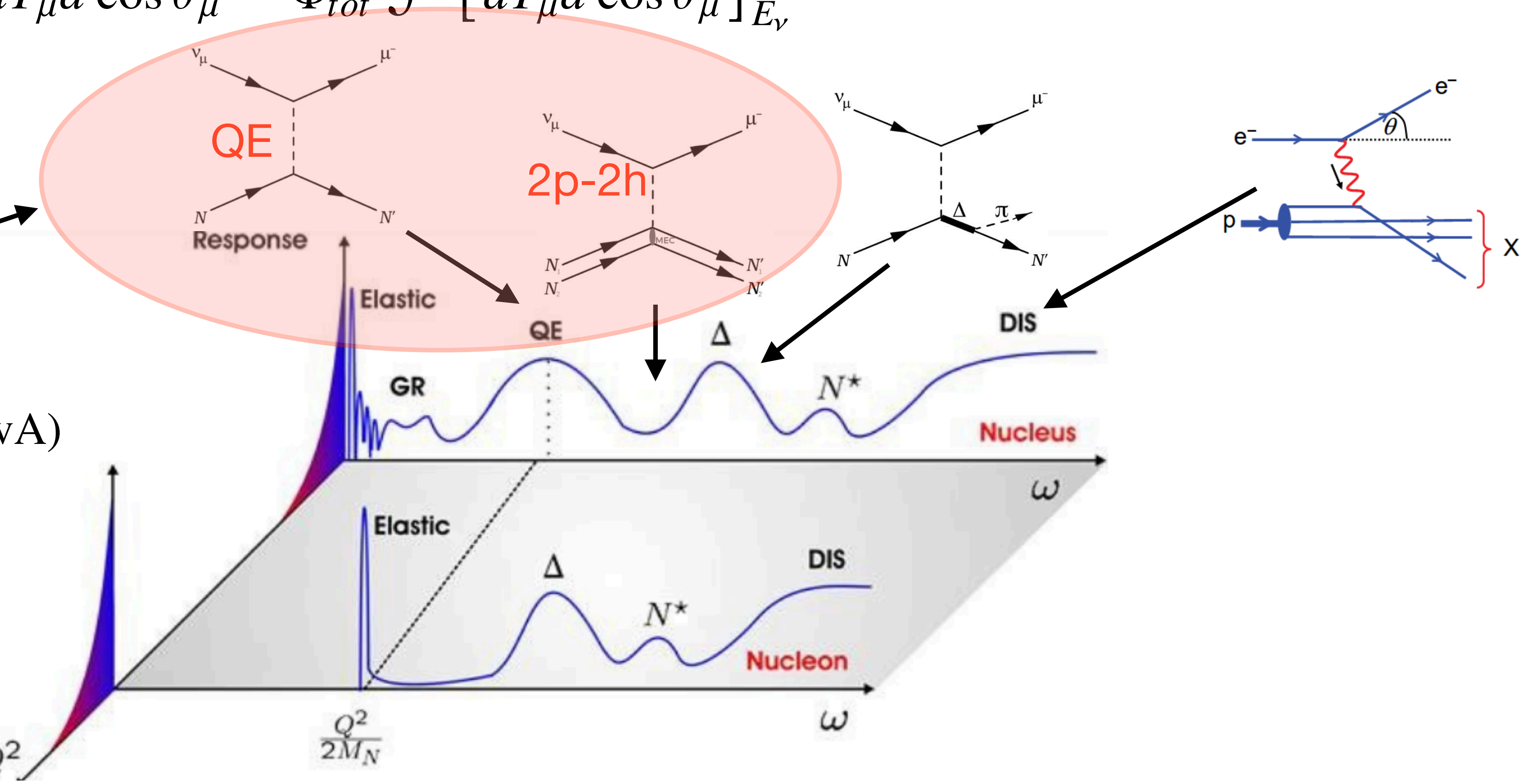
# Nuclear response to electroweak probes

## Neutrino beams are not monochromatic: multi-scale problem

When integrating over the neutrino flux, different processes contribute to the cross section, depending on the energy transferred from the neutrino to the nucleus

$$\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu} = \frac{1}{\Phi_{tot}} \int \left[\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu}\right]_{E_\nu} \Phi(E_\nu) dE_\nu$$

Dominate present neutrino experiments (T2K, MiniBooNE, MINERvA)



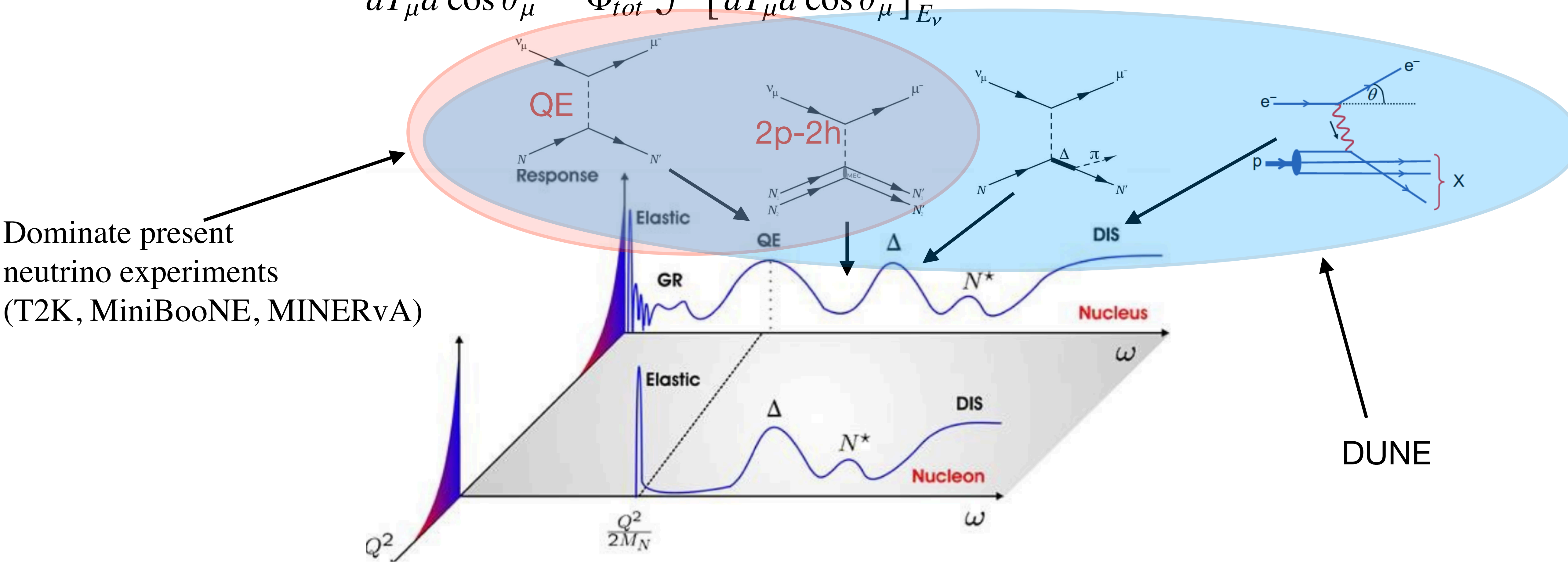

- Quasi-elastic scattering (CCQE): $\nu_l n \to l^- p, \qquad \bar{\nu}_l p \to l^+ n$
- Two-nucleon knockout: $\nu_l NN \to l^- NN, \qquad \bar{\nu}_l NN \to l^+ NN$
- Resonance production: $\nu_l p \to l^- \Delta^{++}, \quad \Delta^{++} \to p\pi^+$ and $\bar{\nu}_l n \to l^+ \Delta^-, \quad \Delta^- \to n\pi^-$
- Deep inelastic scattering $\nu_l/\bar{\nu}_l(k) + N(p) \to \mu^{\mp}(k') + X(p')$
- Coherent meson production $\nu_l + A \to l^- + m^+ + A, \quad \bar{\nu}_l + A \to l^+ + m^- + A$ with $m^\pm = \pi^\pm, K^\pm, \rho^\pm, \ldots$

# Nuclear response to electroweak probes

## Neutrino beams are not monochromatic: multi-scale problem

When integrating over the neutrino flux, different processes contribute to the cross section, depending on the energy transferred from the neutrino to the nucleus

$$\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu} = \frac{1}{\Phi_{tot}} \int \left[\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu}\right]_{E_\nu} \Phi(E_\nu) dE_\nu$$



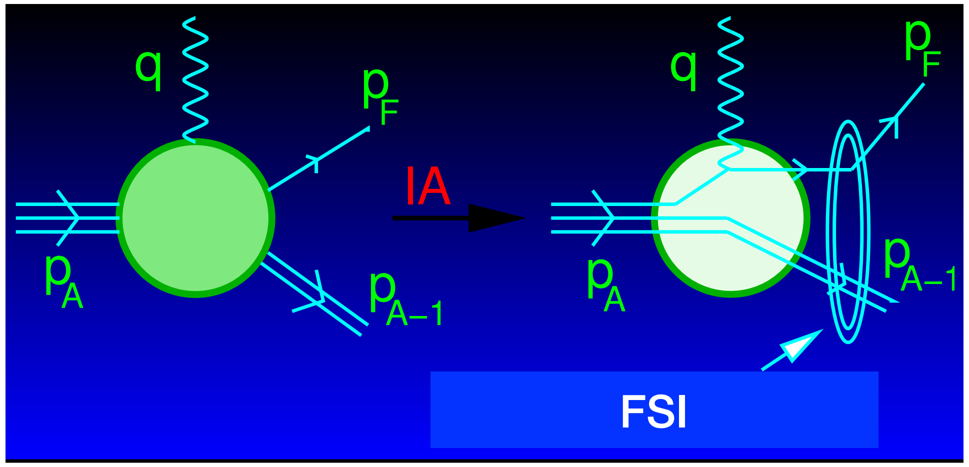

- Quasi-elastic scattering (CCQE): $\nu_l n \to l^- p, \qquad \bar{\nu}_l p \to l^+ n$
- Two-nucleon knockout: $\nu_l NN \to l^- NN, \qquad \bar{\nu}_l NN \to l^+ NN$
- Resonance production: $\nu_l p \to l^- \Delta^{++}, \quad \Delta^{++} \to p\pi^+$ and $\bar{\nu}_l n \to l^+ \Delta^-, \quad \Delta^- \to n\pi^-$
- Deep inelastic scattering $\nu_l/\bar{\nu}_l(k) + N(p) \to \mu^\mp(k') + X(p')$
- Coherent meson production $\nu_l + A \to l^- + m^+ + A, \quad \bar{\nu}_l + A \to l^+ + m^- + A$ with $m^\pm = \pi^\pm, K^\pm, \rho^\pm, \ldots$

## What we want from a nuclear model

Basic requirements for a “good” nuclear model to be used in neutrino oscillation analyses:

- ☑ relativistic
- ☑ compare well with electron scattering data
- ☑ implementable in Monte Carlo generators
- ☐ consistent description of the full spectrum covered by the $\nu$ flux (the most challenging)

## Quasi-elastic region: the Impulse Approximation




**Nuclear Current $\Longrightarrow$ One-body operator**

$$J^{\mu}_{N}(\omega,\vec{q}) = \int d\vec{p}\, \overline{\Psi}_F(\vec{p}+\vec{q})\hat{J}^{\mu}_{N}\Psi_B(\vec{p})$$

**Impulse Approximation:**

scattering off a nucleus = incoherent sum of single nucleon scattering processes

# Quasi-elastic region: the Relativistic Mean Field Model

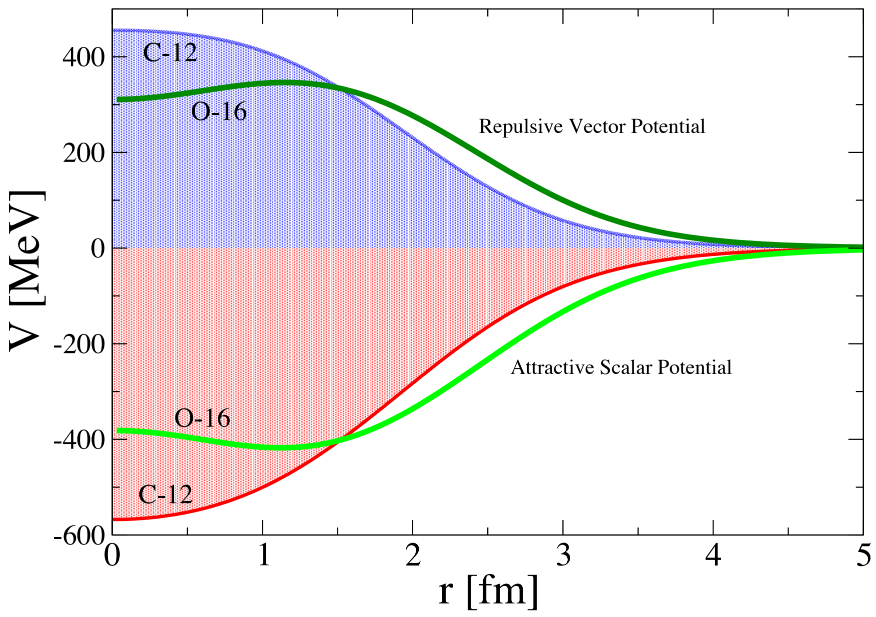
The nucleon wave functions are solutions of the Dirac equation with phenomenological relativistic potentials (scalar and vector)

$$(i\gamma^\mu \partial_\mu - M - S + V)\,\psi(\vec{r}, t) = 0$$

obtained from a Lagrangian fitted to properties of nuclear radii and masses.

**Bound wave function**

$$\Psi_B = \begin{pmatrix} \phi^{up} \\ \phi^{down} \end{pmatrix} = \begin{pmatrix} \phi^{up} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M+S-V}\,\phi^{up} \end{pmatrix} = \alpha u + \beta v$$




The ejected nucleon wave function depends on **final state interactions** (FSI) with the residual nucleus

Different treatments of the ejected nucleon wave function:

- Relativistic Plane Wave Impulse Approximation (**RPWIA**) = no FSI
- FSI can described by:
  - relativistic optical potential, fitted to elastic nucleon-nucleus scattering data: **ROP**
  - the same S & V potentials used for the bound state: **RMF**
    orthogonality preserved: the initial and final wfs are eigenstates of the same H

# FSI in the Relativistic Mean Field Model: superscaling test

What have we learned from inclusive electron scattering $(e,e')$?

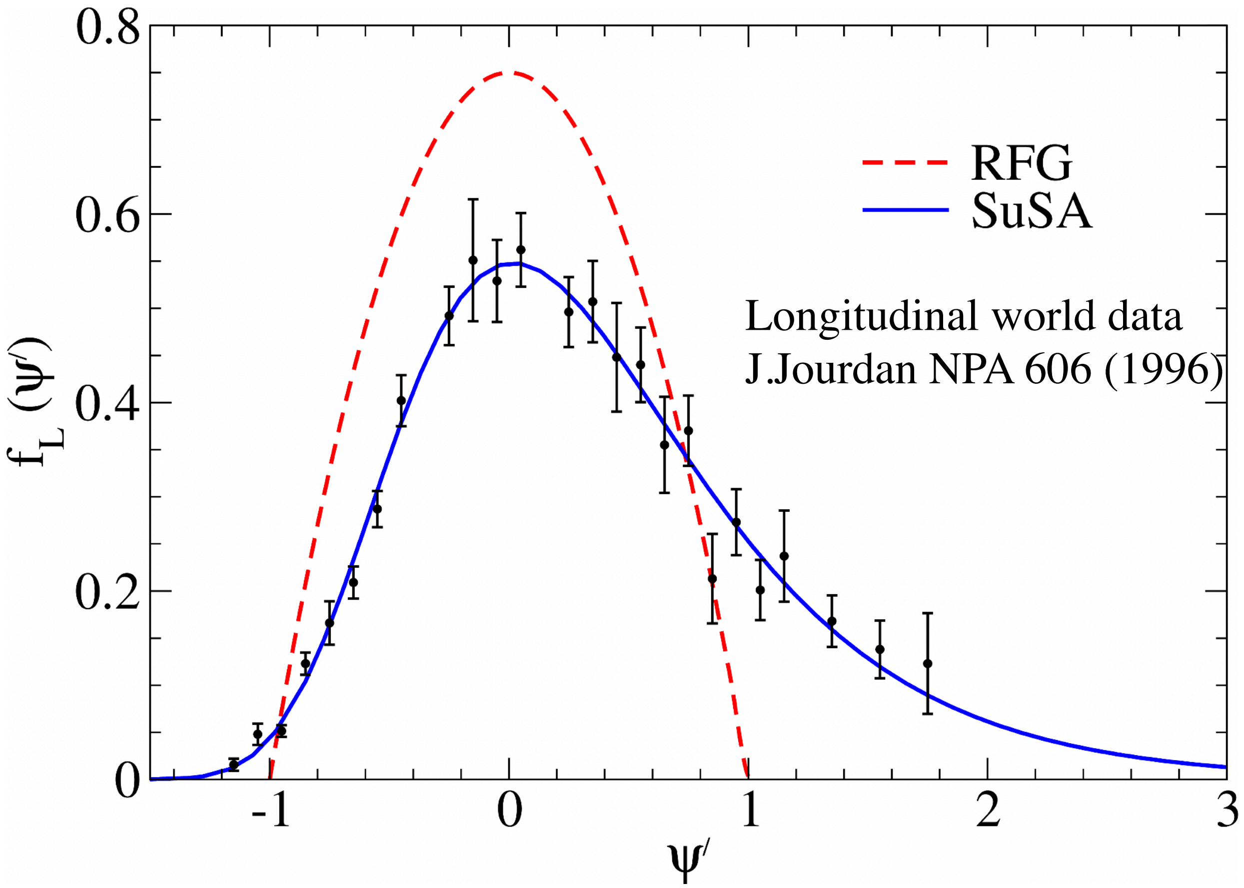
$$f(q,\omega;k_F) \equiv k_F \times \frac{\left[d^2\sigma/d\omega d\Omega_e\right]_{exp}}{\overline{\sigma}_{eN}(q,\omega;p=p_{min},\mathscr{E}=0)} \xrightarrow{q \gtrsim 300\ MeV/c} f(\psi')$$

**The SuperScaling function $f$ extracted from $(e,e')$ QE data at all kinematics and on different nuclei is independent of $q$ (I kind scaling) and of $k_F$ (II kind scaling) [Donnelly and Sick, PRL82 (1999)]**

$f$ embodies the nuclear effects and depends upon only one **scaling variable** (analogous to $x$ in DIS)

$$\psi'(q,\omega) \simeq \frac{m_N}{qk_F}\left(\omega' - \frac{|Q'^2|}{2m_N}\right)$$

$$\omega' = \omega - E_{shift}$$




$$f_L^{RFG}(\psi') = \frac{3}{4}\left(1-\psi'^2\right)\theta\left(1-\psi'^2\right)$$

From L/T separated data it is found that scaling violations mainly occur in the transverse channel at $\psi' > 0$ due to non-QE mechanisms (2p2h, $\Delta$,…)

Stringent constraint on nuclear modelling

# FSI in the Relativistic Mean Field Model: superscaling test

What have we learned from inclusive electron scattering $(e, e')$?

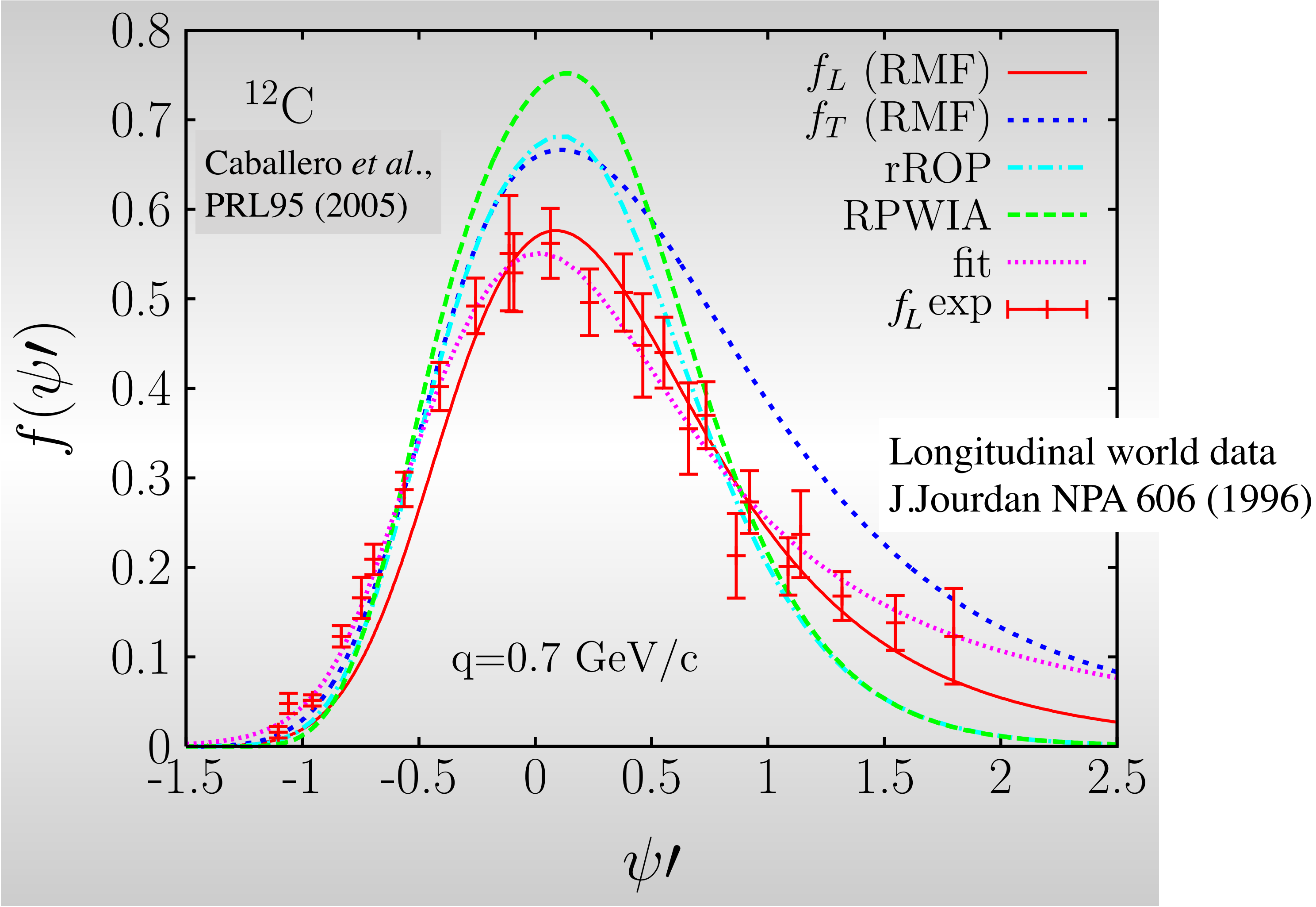
$$f(q,\omega;k_F) \equiv k_F \times \frac{\left[d^2\sigma/d\omega d\Omega_e\right]_{exp}}{\overline{\sigma}_{eN}(q,\omega;p=p_{min},\mathcal{E}=0)} \xrightarrow{q \gtrsim 300\ MeV/c} f(\psi')$$

**The SuperScaling function $f$ extracted from $(e, e')$ QE data at all kinematics and on different nuclei is independent of $q$ (I kind scaling) and of $k_F$ (II kind scaling) [Donnelly and Sick, PRL82 (1999)]**

$f$ embodies the nuclear effects and depends upon only one **scaling variable** (analogous to $x$ in DIS)

$$\psi'(q,\omega) \simeq \frac{m_N}{qk_F}\left(\omega' - \frac{|Q'^2|}{2m_N}\right)$$

$$\omega' = \omega - E_{shift}$$




Only the description of FSI provided by RMF leads to an asymmetric scaling function in accordance with the behaviour shown by (e,e') data.

Moreover it is the only model to provide

$$f_T > f_L$$

in agreement with L/T separated data.

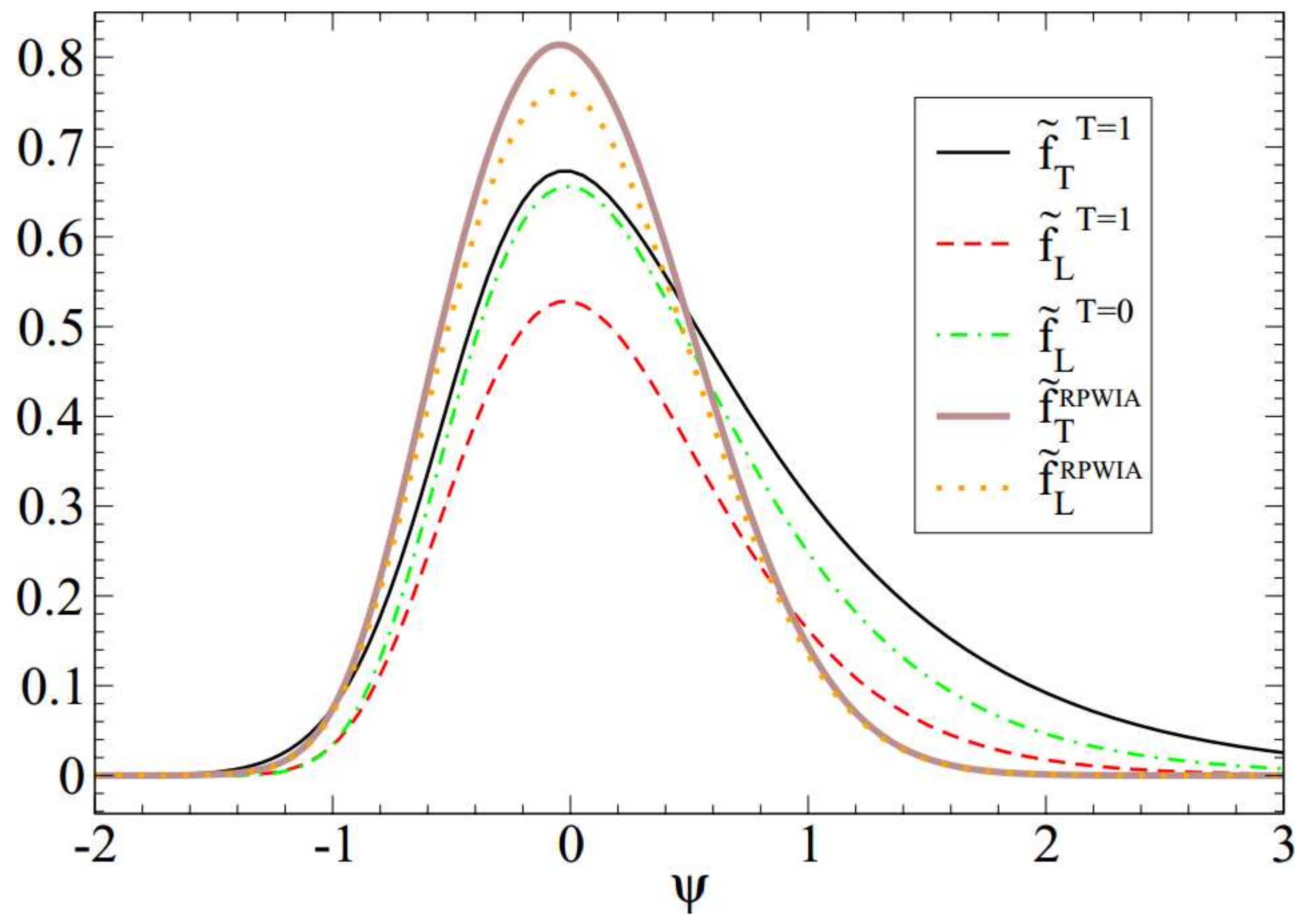
# The “SuSAv2” Model

Gonzalez et al., PRC90(2014)
Megias et al., PRD94(2016)

- In the “**SuSAv2**” model the weak response functions are evaluated as

$$R^K(q,\omega) = k_F \times G^K(q,\omega) \times f^K(\psi')$$

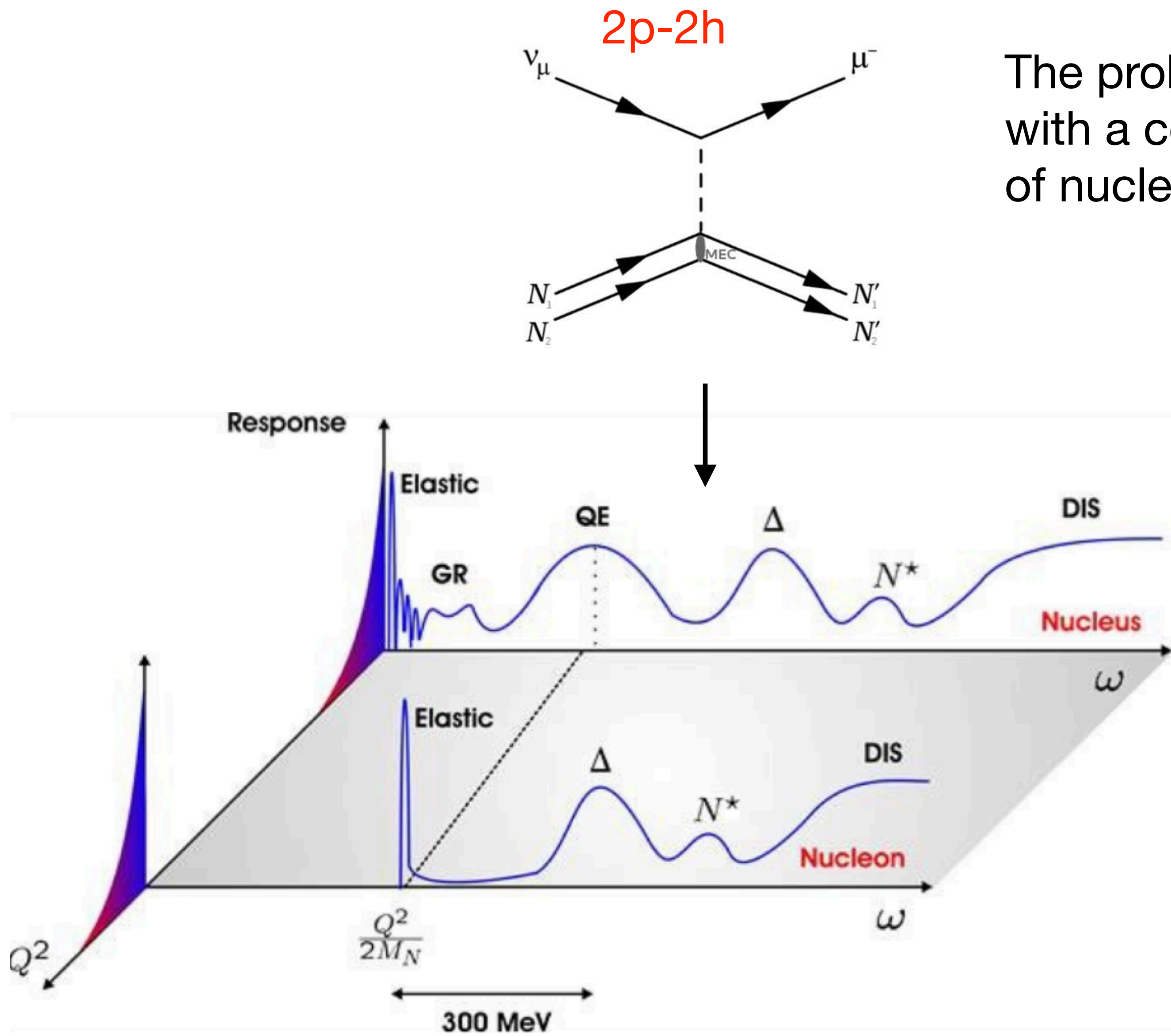
$G^K$ single-nucleon responses
$f^K$ **set of scaling functions**, given by the **RMF** calculation for all reaction channels (L/T, isovector/isoscalar, V/A)




- The shortcoming of the RMF of being too strong at high energies is corrected for by introducing a q-dependent blending function which **mixes RMF and RPWIA final states**. This introduces two parameters, which are fitted once and for all to Carbon (e,e’) data.

# Beyond the Impulse Approximation:
# Meson Exchange Currents and 2p2h excitations

The probe interacts with a correlated pair of nucleons: beyond IA

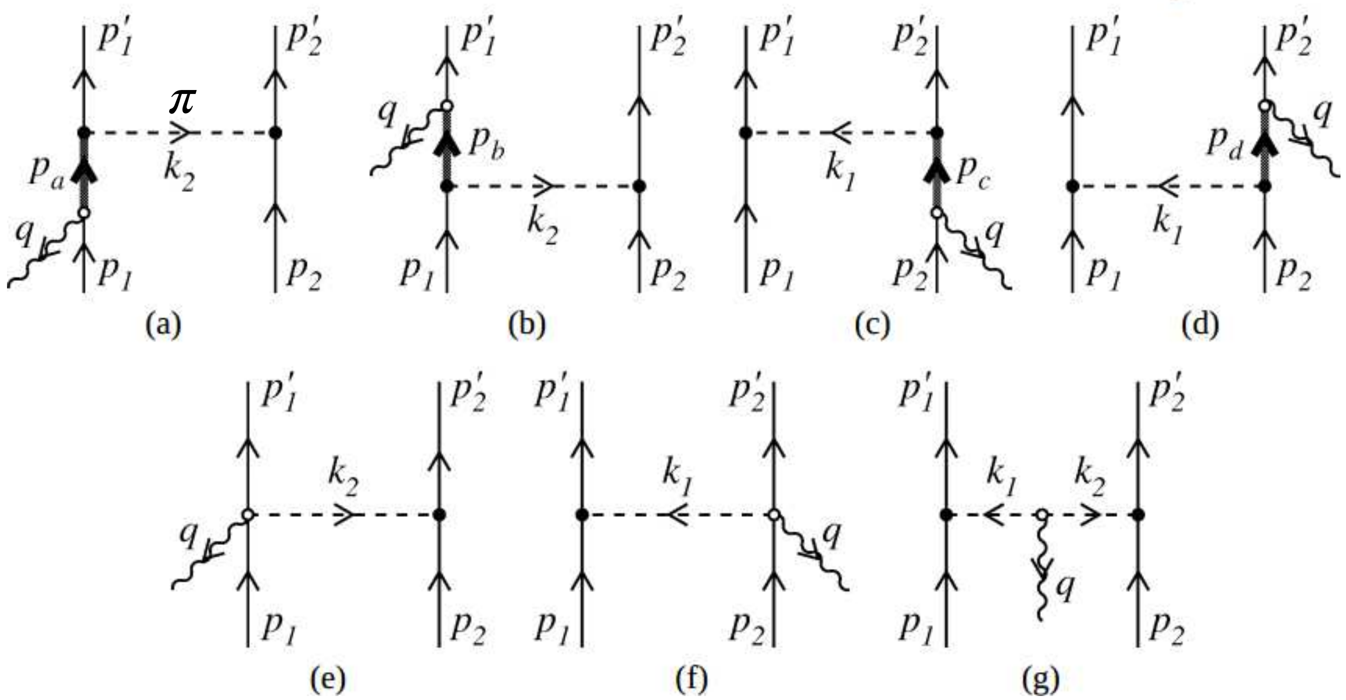


# Meson Exchange Currents

De Pace et al., Nucl.Phys. A726 (2003) 303-326 — electromagnetic MEC
Ruiz Simo et al., J.Phys. G44 (2017) no.6, 065105 — extension to weak sector

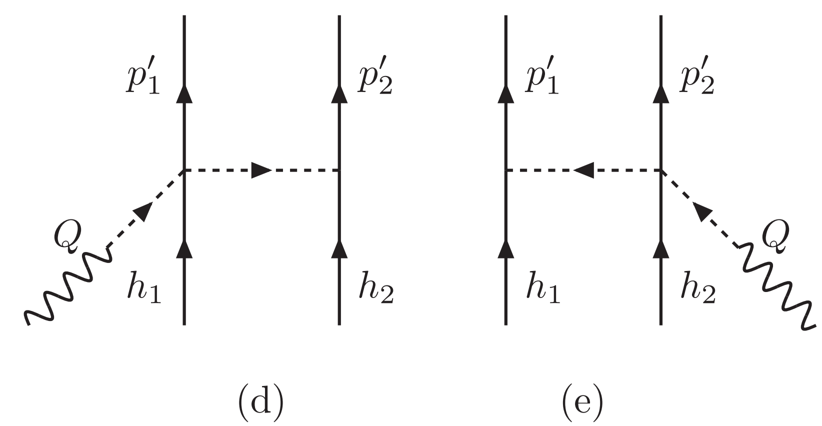
Two-body currents in **free space**

"Δ pole"




"Seagull" or "contact"    "Pion in flight"

"Pion pole"
(only for neutrinos, purely axial)




**In the medium**, they give rise to a huge amount of many-body diagrams, corresponding to the excitation of **2p2h** states. In the RFG the corresponding hadronic tensor is:

$$W^{\mu\nu}_{\text{2p-2h}} = \frac{V}{(2\pi)^9} \int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \times r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2)\delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \times \Theta(p'_1, p'_2, h_1, h_2)\delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{h}_1 - \mathbf{h}_2 - \mathbf{q}),$$

$$\Theta(p'_1, p'_2, h_1, h_2) = \theta(p'_2 - k_F)\theta(p'_1 - k_F)\theta(k_F - h_1)\theta(k_F - h_2).$$

$$r^{\mu\nu}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2) = \frac{1}{4}\sum_{s,t} j^\mu(1', 2', 1, 2)^*_A j^\nu(1', 2', 1, 2)_A,$$

7-dimensional integral

# 2p2h MEC many-body diagrams

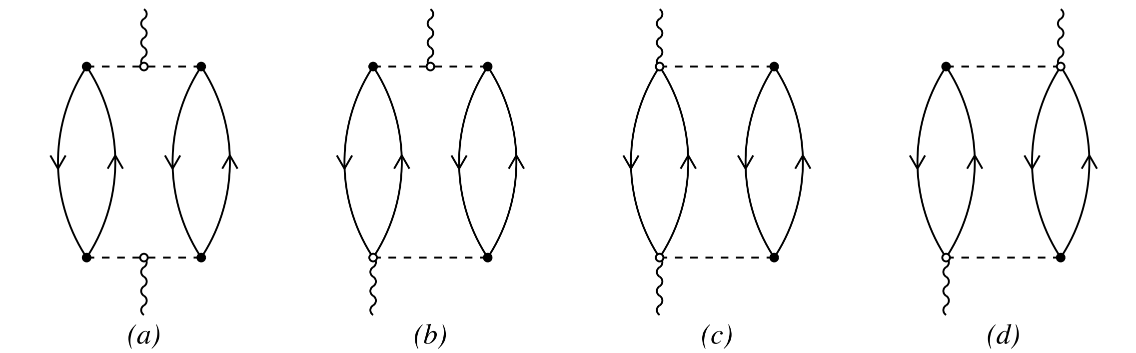



FIG. 2: The direct pionic contributions to the MEC 2p-2h response function.

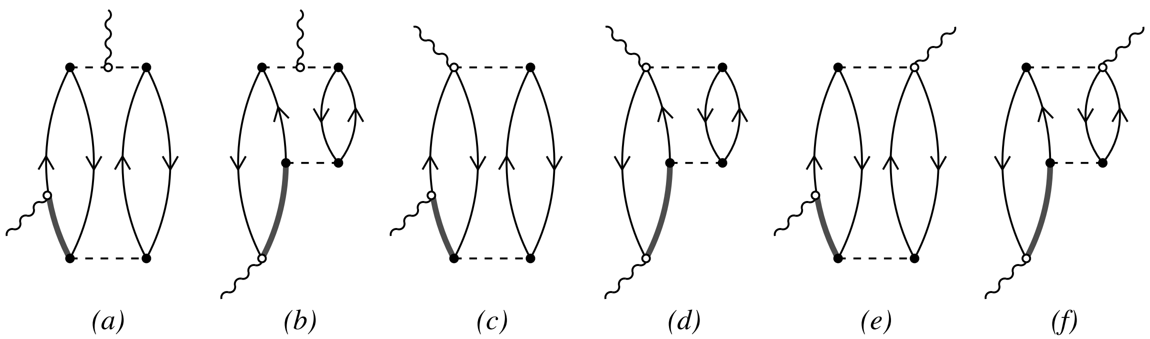



FIG. 3: The direct pionic/Δ interference contributions to the MEC 2p-2h response function.

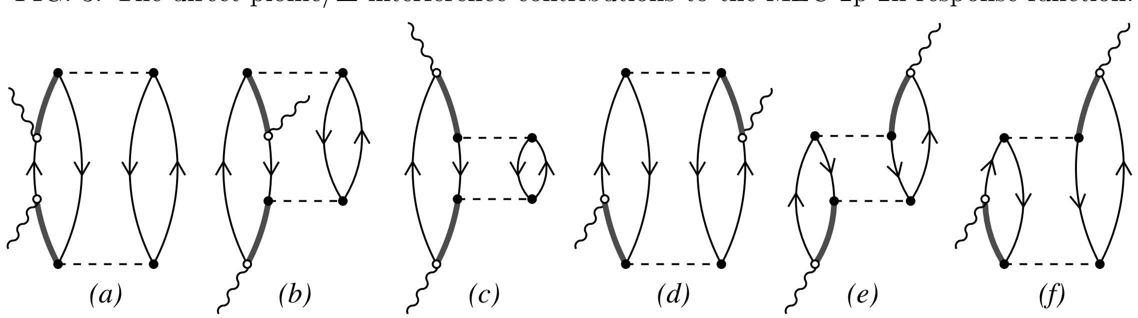



FIG. 4: The direct Δ contributions to the MEC 2p-2h response function.

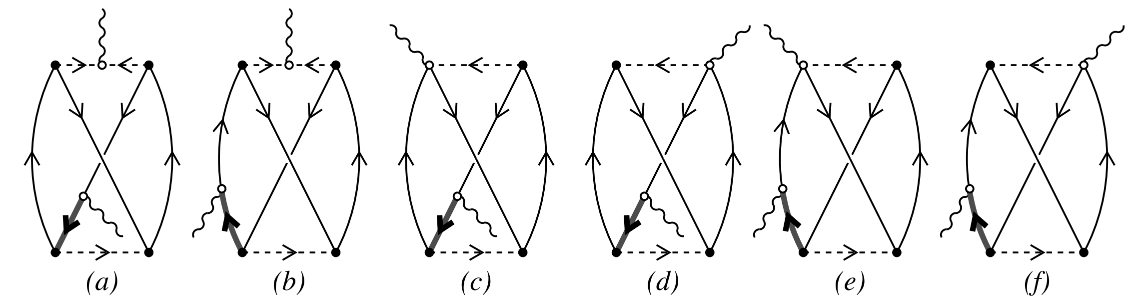



FIG. 5: The exchange pionic/Δ interference contributions to the MEC 2p-2h response function.

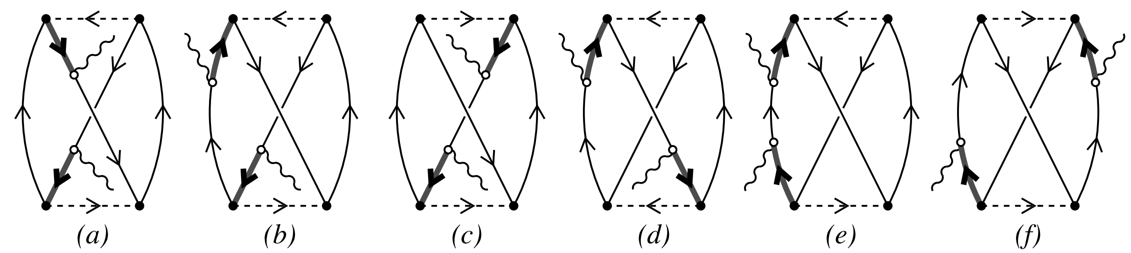



FIG. 6: The exchange Δ contributions to the MEC 2p-2h response function.

**De Pace et al., NPA726 (2003)**

- **fully relativistic** calculation **based on RFG**
- all many-body diagrams involving 2 pions are included
- each diagram is a 7-dimensional integral+flux integration
- the numerical calculation has been checked using two different techniques:

1. polarization propagator, many-body Goldstone diagrams, analytical manipulation of isospin traces and Dirac matrices spin traces using FORM, Monte Carlo integration
2. numerical evaluation of the hadronic tensor, including spin and isospin traces: **np, nn and pp can be separated**

Some representative results:

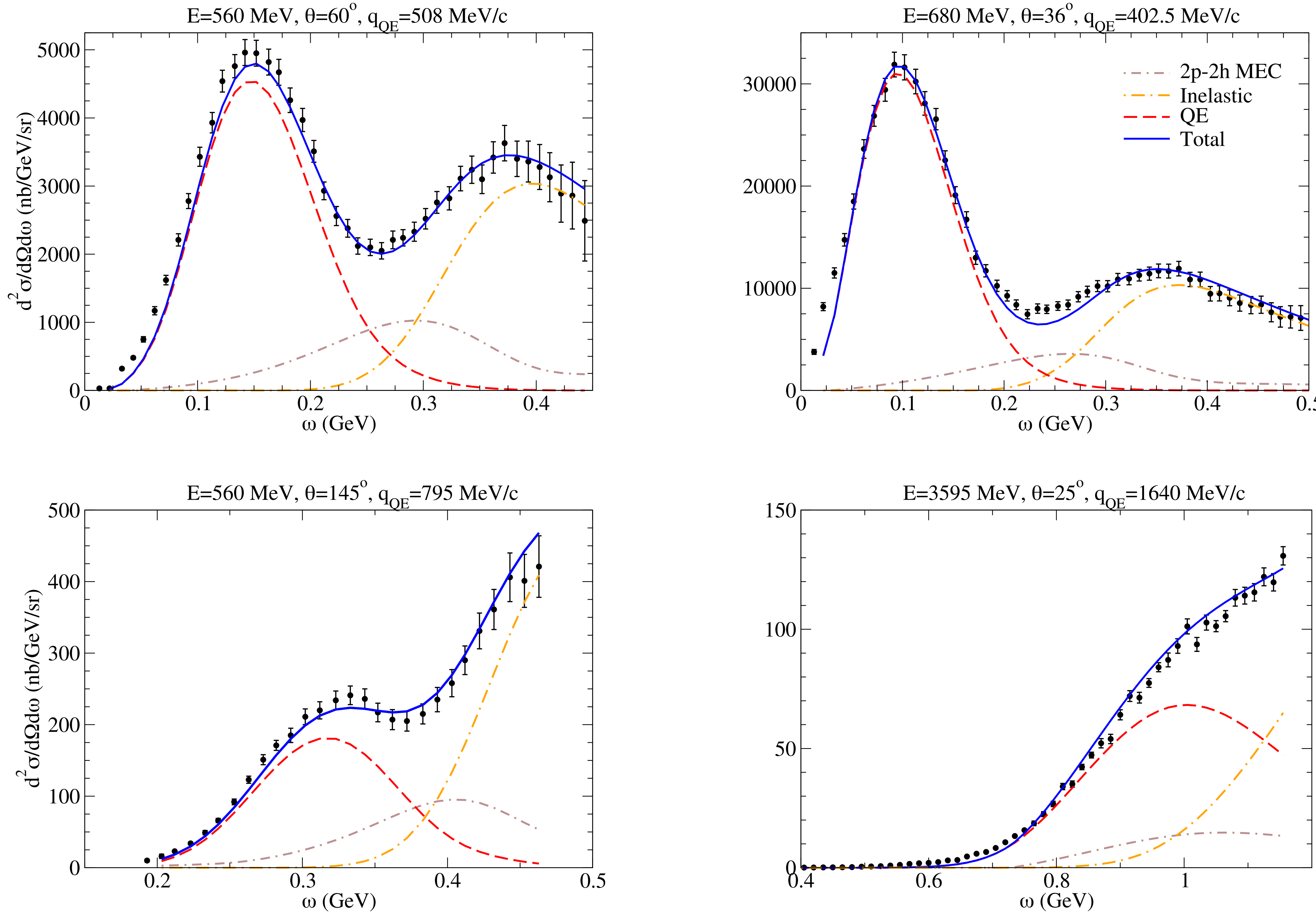
1. $A(e, e')X$ inclusive electron scattering

2. $A(\nu_\mu, \mu)X$ charged current neutrino scattering

"CCQE-like" or "CC0$\pi$"

no pions in the final state

# Validation: Carbon (e,e')

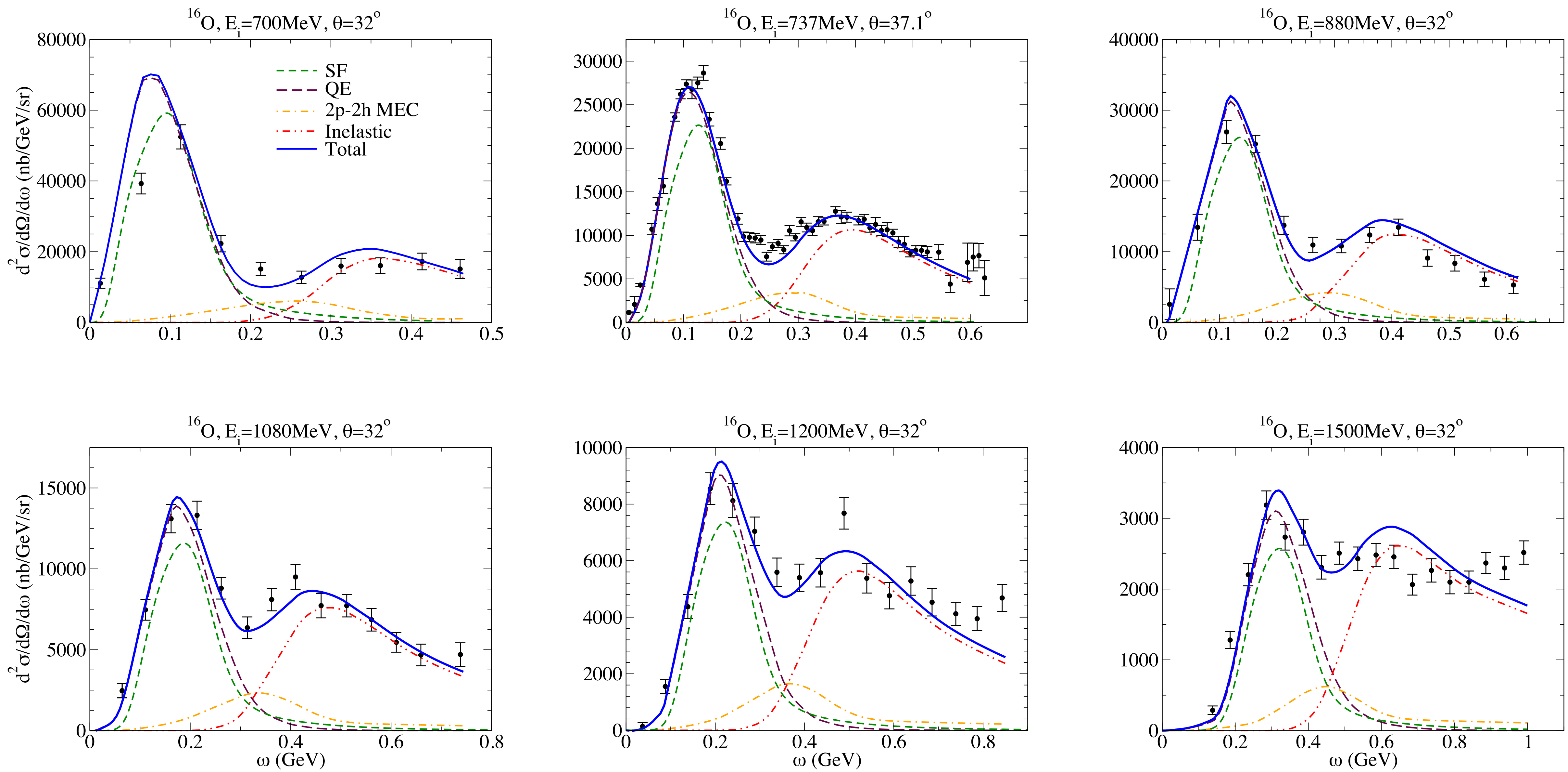



G.D. Megias *et al*., PRD94 (2016)

Data: Barreau, NPA 402A (1983)
Day. PRC 48 (1993)

Good agreement with data
in a wide kinematical region

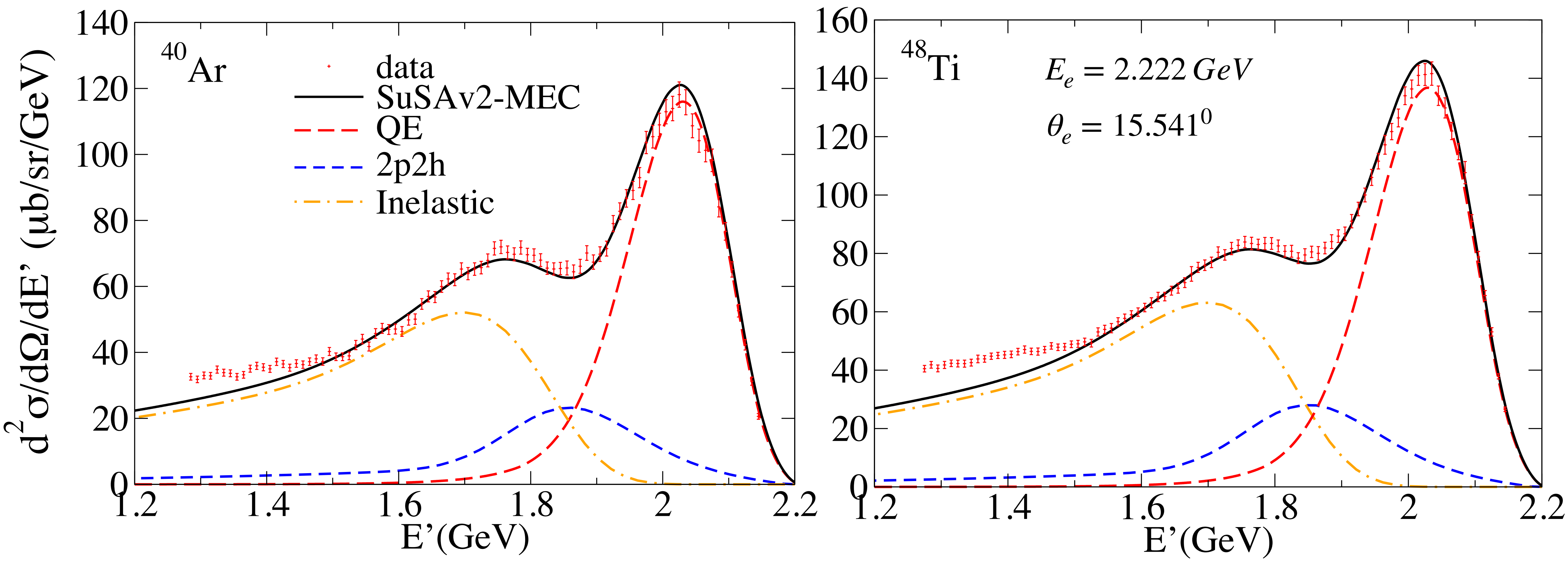
# Validation: Oxygen (e,e')




G.D. Megias *et al.*, JPG 46 (2019)

Data: Anghinolfi *et al.*, NPA 602 (1996)
O'Connell *et al.*, PRC 35 (1987)

# Validation: JLab (e,e') data on Ar and Ti

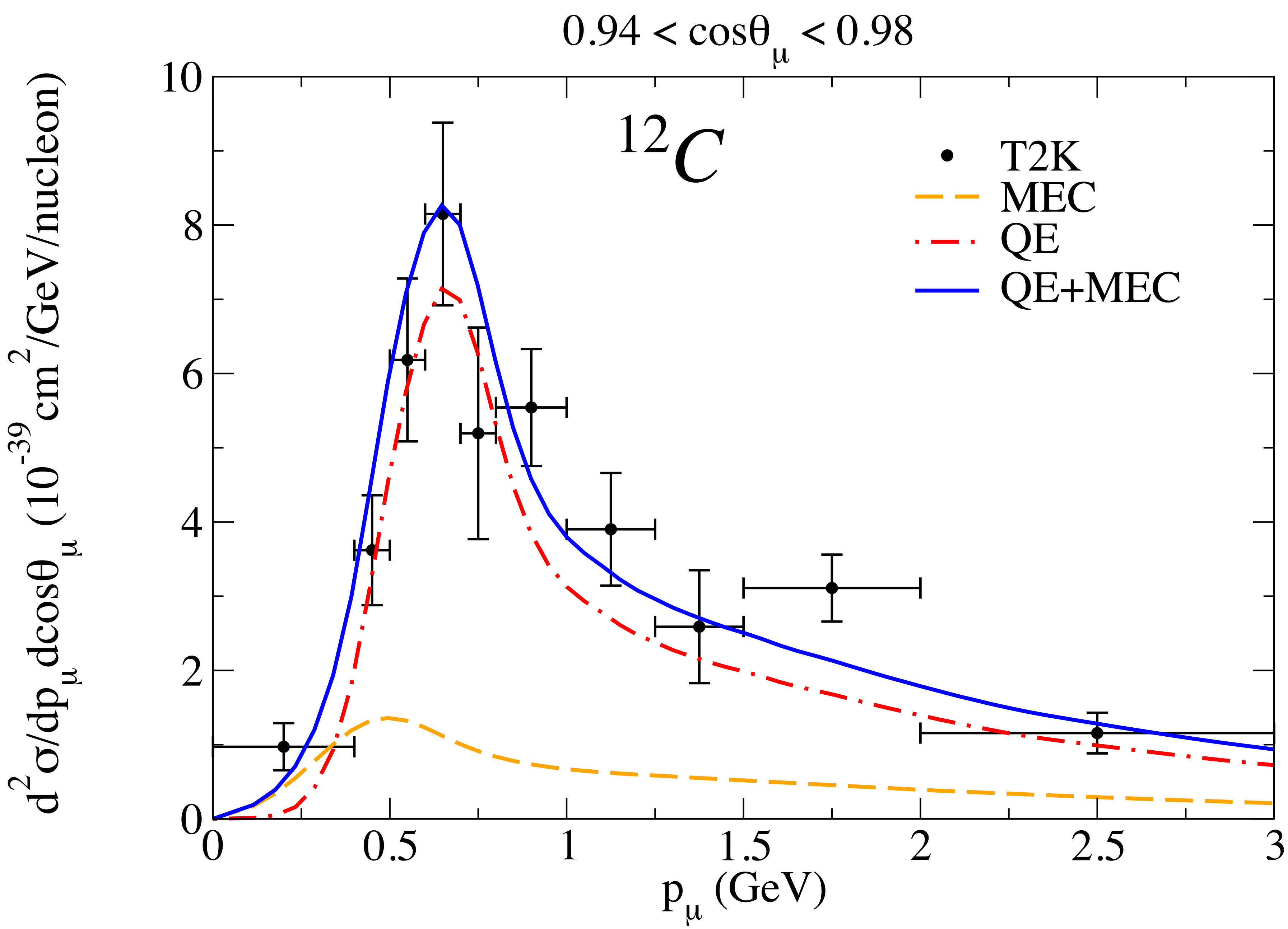



Data from H.Dai *et al*., PRC98 (2018); PRC99 (2019)
Experiment aimed at measuring the Argon spectral function

MBB *et al*., PRC99 (2019)

## T2K $(\nu_\mu, \mu^-)$ $CC0\pi$

no pions in the final state

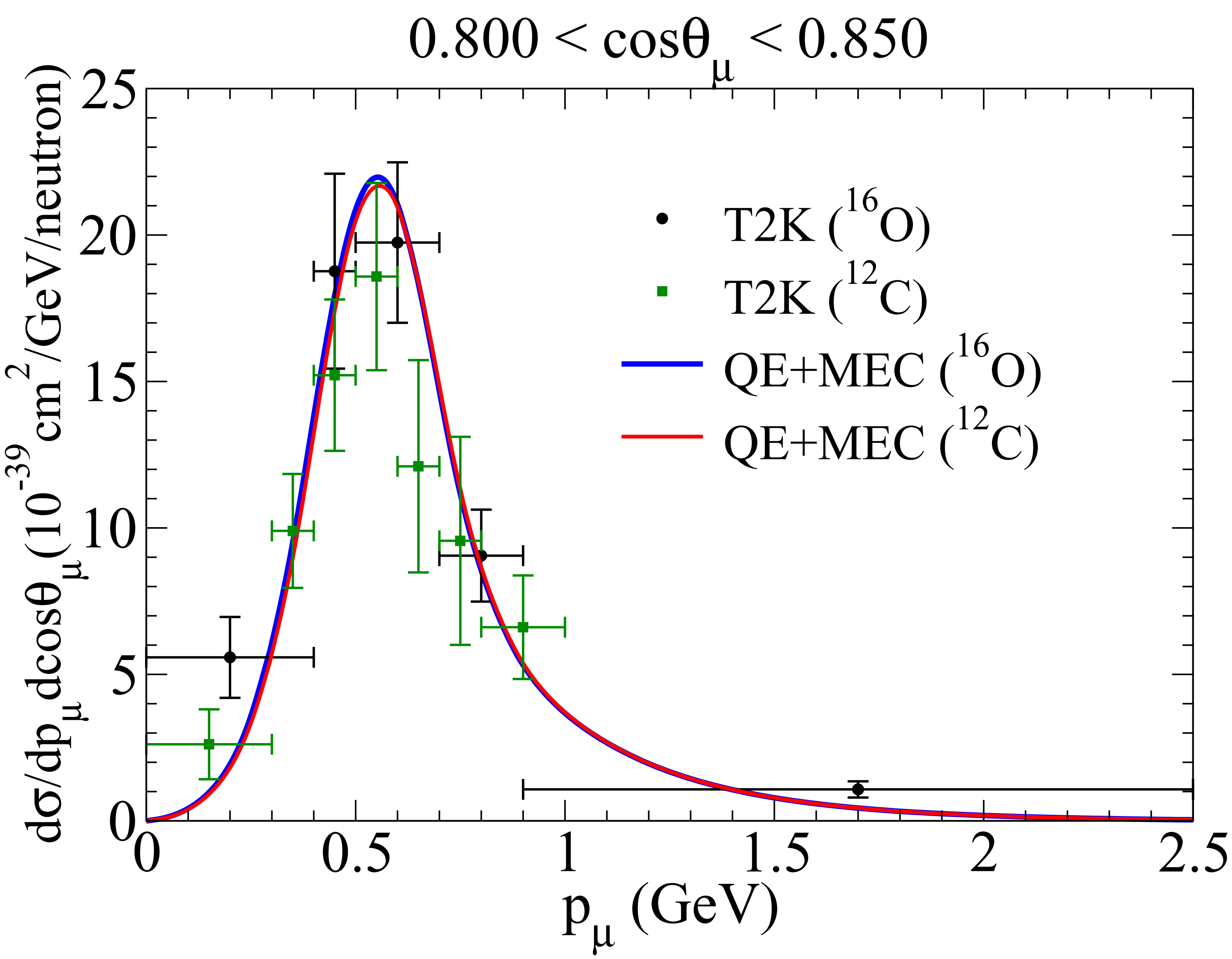



$CC0\pi$ data

T2K data, PRD 97 (2018)

G.D. Megias et al, JPG46 (2019)

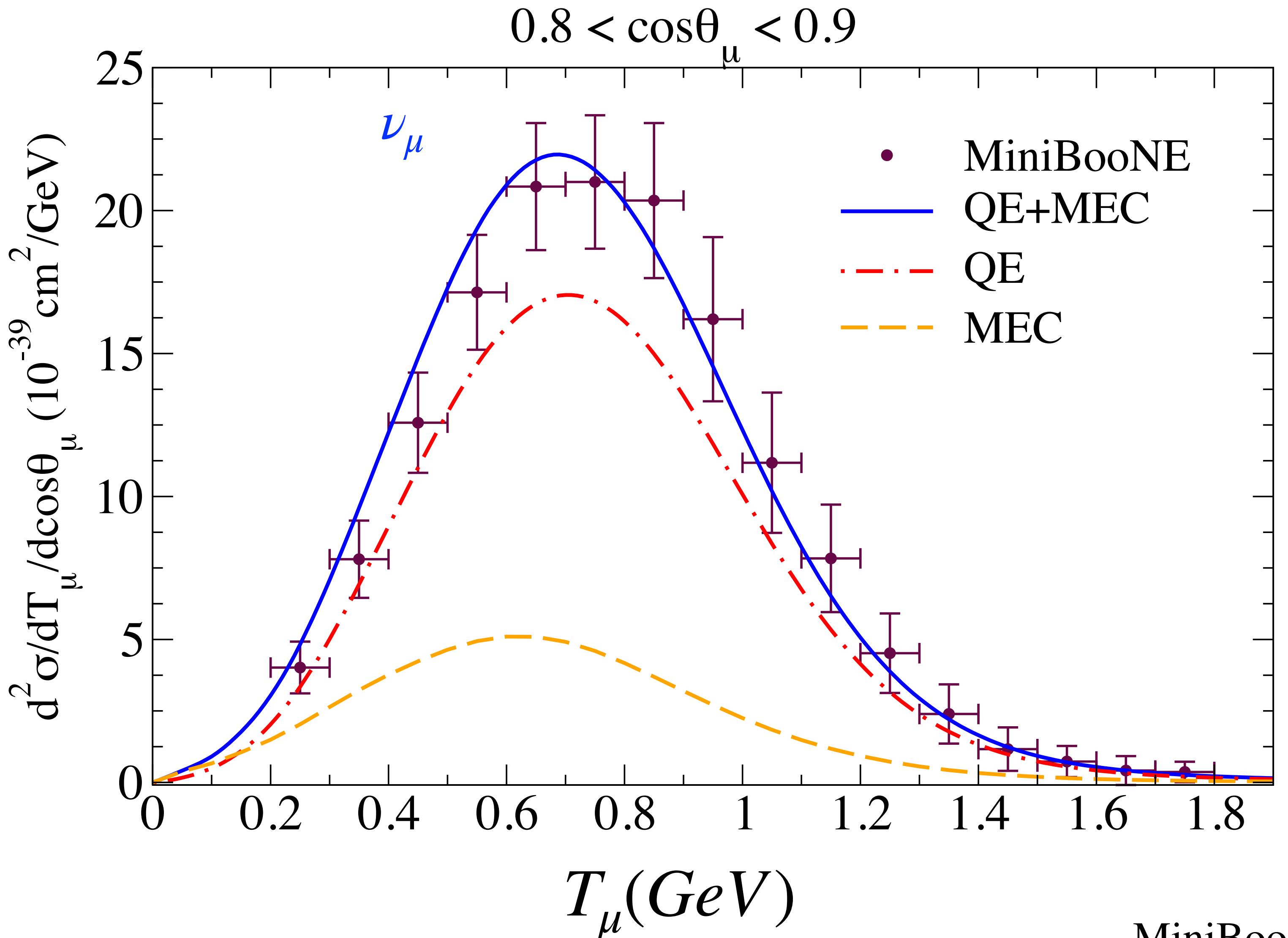
# T2K: Oxygen versus Carbon, $CC0\pi$




Megias *et al*., JPG 46 (2019)

Negligible differences between oxygen and carbon at T2K conditions

MiniBooNE $(\nu_\mu, \mu^-)$ CCQE-like
no pions in the final state




G.D. Megias *et al*.,
PRD94 (2016)

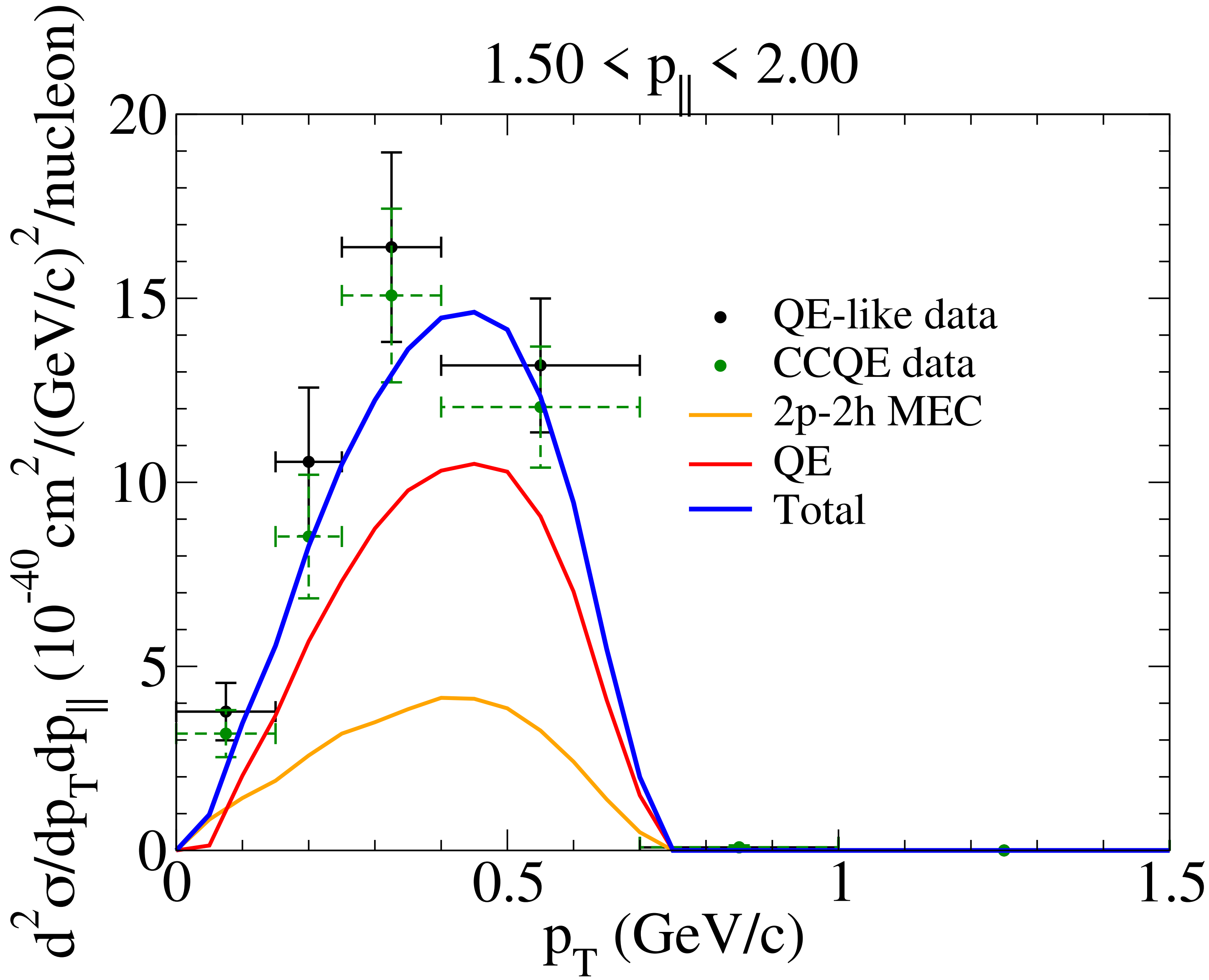
MiniBooNE data
Phys. Rev. D81 (2010)

MINERvA $(\bar{\nu}_\mu, \mu^+)$ $CC0\pi$

no pions in the final state

$p_\parallel = p_\mu \cos\theta_\mu$

$p_\perp = p_\mu \sin\theta_\mu$



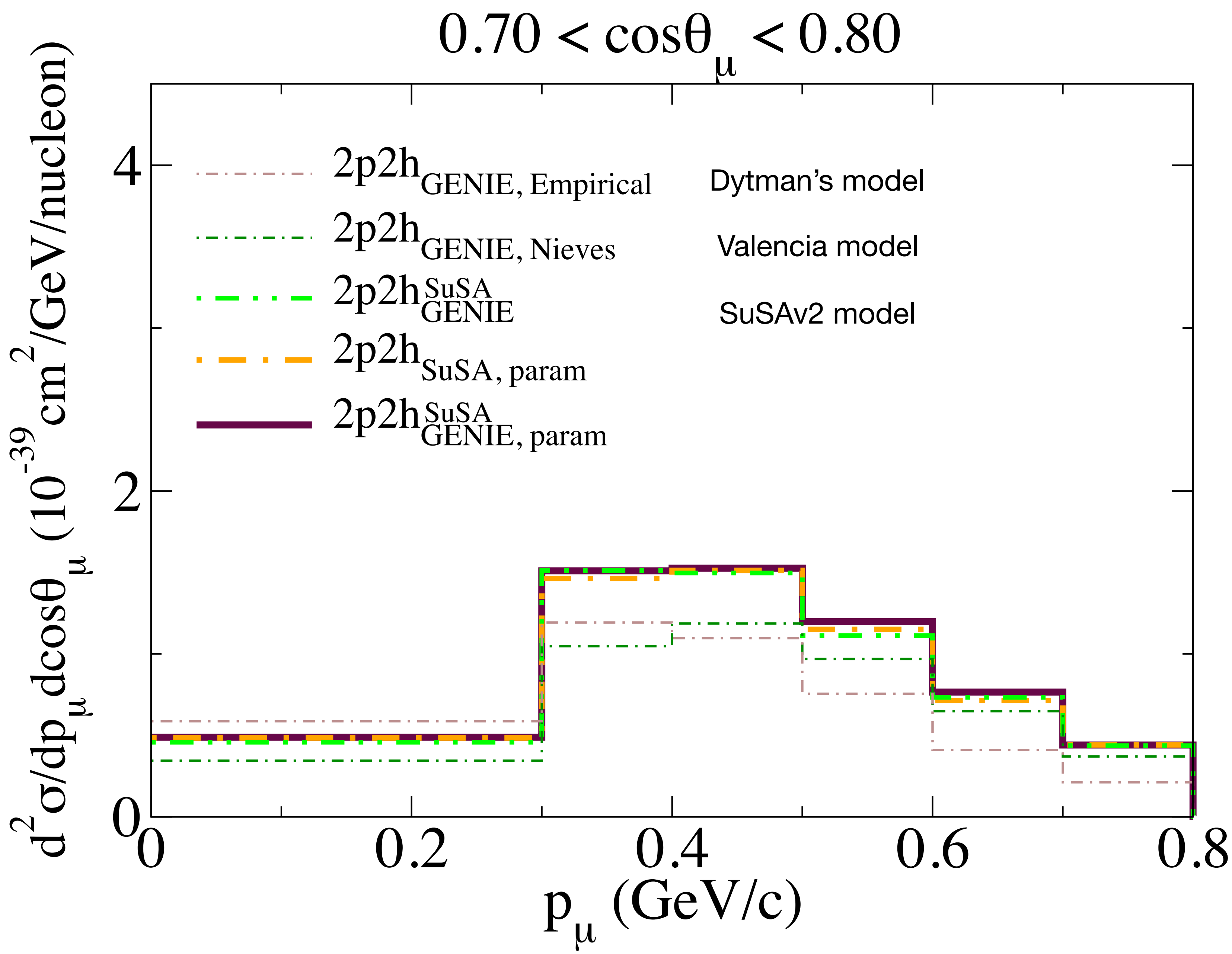

$\bar{\nu}_\mu - CH$

Megias *et al.*,
PRD 99 (2019)

MINERvA data
PRD 97 (2018)

# Implementation of SuSAv2 in GENIE (S.Dolan, G.Megias, S.Bolognesi)

Recent effort for implementing the 2p2h model in GENIE
Comparison with other models:




S.Dolan *et al.*,
arXiv:1905.08556

**Important differences between 2p2h models implemented in GENIE**
**Work is progress to implement the 1p1h model**

# Summary and future work

- Validation against (e,e’) data is a solid benchmark for nuclear models to be used in analyses of neutrino oscillation experiments. Superscaling sets strong constraints on modelling.

- The SuSAv2-MEC model, based on RMF and including FSI and two-body currents, provides a satisfactory comparison with both electron and neutrino scattering off different nuclei (carbon, oxygen, calcium, titanium, argon).

- Computationally demanding microscopic calculations can be translated into a rather straightforward formalism, easier to be implemented in MC event generators (GENIE: 2p2h implemented, 1p1h in progress).

- Next steps:

  - inclusive neutrino scattering including all inelasticities (DIS), important for DUNE

  - semi-inclusive reactions (more sensitive to nuclear model effects), necessary to compare with future exclusive measurements and to get more reliable implementation in MC generators.
    RMF has been successfully used in the 90s to describe $(e, e'p)$.
    Extension to weak reactions is in progress.

# Collaboration

- **M.B.B., A. De Pace (Torino)**
- **J.A. Caballero (Sevilla), G. Megias (Sevilla/Saclay)**
- **J.E. Amaro, I. Ruiz Simo (Granada)**
- **T.W. Donnelly (MIT)**
- **J.M. Udias, R. Gonzalez-Jimenez (Madrid)**
- **A. Antonov, M. Ivanov (Sofia)**
- **W. Van Orden (ODU & JLab)**

**Suggested reading: NuSTEC White Paper, Prog.Part.Nucl.Phys. 100 (2018)**

***THANK YOU***