Is there a comprehensive picture of nuclear SRC?  
(quest to learn about stylized facts of nuclear SRC)

How to forge links between nuclear-structure theory and A(e, e′pX) observables sensitive to nuclear SRC?  
(forging bridges between theories of “nuclear structure” and of “nuclear reactions”)
1. Low-order correlation operator approximation (LCA) to compute effect of SRC (nuclear structure & nuclear reactions)

2. Apply LCA to the computation of nuclear momentum distributions (NMDs) for 15 nuclei $A(N,Z): 4 \leq A \leq 208$ and $1 \leq \frac{N}{Z} \leq 1.54$

   **CHECK:** Compare LCA results to ab-initio ones

3. Aggregated effect of SRC and its evolution with $A$ and $N/Z$

   **CHECK:** $a_2$ data from $A(e, e')$

4. Isospin composition ($pp&nn&pn$) of SRC

   **CHECK:** $A(e, e'pp), A(e, e'pn), A(e, e'p)$ data for $^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb$ in “SRC” kinematics

5. $N/Z$ asymmetry dependence of SRC

   **CHECK:** $A(e, e'pp), A(e, e'pn), A(e, e'p), A(e, e'n)$ data for $^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb$ in “SRC” kinematics
Single-nucleon momentum distributions in LCA

- Single-nucleon momentum distribution $n^{[1]}(p)$

$$n^{[1]}(p) = \frac{A}{(2\pi)^3} \int d^2 \Omega_p \int d^3 \vec{r}_1 \ d^3 \vec{r}_1' \ d^3 (A^{-1}) \{\vec{r}_2 - A\} \times e^{-i\vec{p} \cdot (\vec{r}_1' - \vec{r}_1)} \Psi^*(\vec{r}_1, \vec{r}_2 - A) \psi(\vec{r}_1', \vec{r}_2 - A)$$

- Universal correlation operators

$$|\Psi\rangle = \hat{G} |\Phi\rangle / \sqrt{\langle \Phi | \hat{G}^\dagger \hat{G} | \Phi \rangle} ,$$

- $\hat{G}$: Central $g_c(r)$, spin-isospin $f_{\sigma\tau}(r)$, tensor $f_{t\tau}(r)$ correlations

- Truncation at $O(G^2)$: SRC part of $n^{[1]}(p) = 2$-body contributions

- Quantify the $pp$, $nn$, $pn$ and $np$ contribution to $n^{[1]}(p)$

(Ghent University)  N to Z dependence of SRC  INPC 2019, Glasgow
Two distinct momentum regimes ("IPM" and "SRC")

Momentum dependence of fat tail of $n^{[1]}$ is "universal"

Probability distribution $P(p) \sim p^2 n^{[1]}(p)$

(Nucleon Momentum $p$ [fm$^{-1}$])

- $^4$He
- $^{16}$O
- $^{40}$Ca
- $^{208}$Pb
Probability distribution $P(p) \sim p^2 n^{[1]}(p)$
Probability distribution \( P(p) \sim p^2 n^{[1]}(p) \)

![Graph showing probability distribution for different channels in 208Pb]
Ratios of probability distributions: $P^A(p)/P^d(p)$


Proton part: $P^A_{pp}(p)$

Neutron part: $P^A_{nn}(p)$

$N=Z$: $^4$He, $^{12}$C, $^{16}$O, $^{40}$Ca

$N\neq Z$: $^9$Be, $^{27}$Al, $^{40}$Ar, $^{48}$Ca, $^{56}$Fe, $^{63}$Cu, $^{84}$Kr, $^{108}$Ag, $^{124}$Xe, $^{197}$Au, $^{208}$Pb
Ratios of probability distributions: $P^A(p)/P^d(p)$

$$P^A(p) = P^A_{pp}(p) + P^A_{pn}(p) + \underbrace{P^A_{nn}(p) + P^A_{np}(p)}_{P^A_n(p) \text{ (neutron part)}}.$$ 

$P^A_{pp}(p) \text{ (proton part)}$

**N=Z:** $^4\text{He}, \ ^{12}\text{C}, \ ^{16}\text{O}, \ ^{40}\text{Ca}$

**N≠Z:** $^9\text{Be}, \ ^{27}\text{Al}, \ ^{40}\text{Ar}, \ ^{48}\text{Ca}, \ ^{56}\text{Fe}, \ ^{63}\text{Cu}, \ ^{84}\text{Kr}, \ ^{108}\text{Ag}, \ ^{124}\text{Xe}, \ ^{197}\text{Au}, \ ^{208}\text{Pb}$
$\alpha_2(A/^{2}H)$ from $A(e, e')$ at $x_B \gtrsim 1.5$ and LCA

Aggregated quantitative effect of SRC in A relative to d

$$\alpha_2(A) = \frac{\int_{p > 2 \text{ fm}^{-1}} dp P^A(p)}{\int_{p > 2 \text{ fm}^{-1}} dp P^d(p)} ; \quad \alpha_2^{exp}(A) = \frac{2 \sigma^A(e,e')}{A \sigma^d(e,e')} \quad (1.5 \lesssim x \lesssim 1.9 ; \quad Q^2 \approx 2 \text{ GeV}^2)$$

1. $A \lesssim 27$: soft A dependence
2. $A \gtrsim 27$: SATURATION
3. Ca isotopes: $\alpha_2(^{40}\text{Ca}) \approx \alpha_2(^{48}\text{Ca})$

$a_2(A/^{2H})$ from $A(e, e')$ at $x_B \gtrsim 1.5$ and LCA

Aggregated quantitative effect of SRC in $A$ relative to $d$

\[ a_2(A) = \frac{\int_{p>2 \text{ fm}^{-1}} dp P_A(p)}{\int_{p>2 \text{ fm}^{-1}} dp P_d(p)} ; \quad a_2^{\text{exp}}(A) = \frac{2}{A} \frac{\sigma^A(e,e')}{\sigma^d(e,e')} \quad (1.5 \lesssim x \lesssim 1.9 ; \quad Q^2 \approx 2 \text{ GeV}^2) \]

Nuclear momentum distribution: pair composition

Pair composition: \( n^{[1]}(p) \equiv n^{[1]}_{pp}(p) + n^{[1]}_{pn}(p) + n^{[1]}_{nn}(p) + n^{[1]}_{np}(p) \)

- SRC pair fractions

\[ r_{pp}(p) = \frac{n^{[1]}_{pp}(p)}{n^{[1]}(p)} \]

- \( r_{nn} \) are momentum dependent


(Ghent University)
Pair composition of SRC: LCA versus experiment

LCA: Ratios from computed $n^\text{[1]}(p)$ for 15 nuclei

\[
\frac{\int_{p_l}^{p_h} dp \; p^2 \; n^\text{[1]}_{pp}(p)}{\int_{p_l}^{p_h} dp \; p^2 \; \left[ n^\text{[1]}_{pn}(p) + n^\text{[1]}_{np}(p) \right]}
\]

M. Duer et al., PRL122(2019): Ratios from measured

\[
\frac{\sigma_{en} \; A(e, e'pp)}{2\sigma_{ep} \; A(e, e'n\bar{p})} \bigg|_{p_l \leq p_m \leq p_h}
\]

for $A=^{12}\text{C, 27Al, 56Fe, 208Pb}$
Fourth moment of $n^{[1]}(p)$ from LCA

Fourth moment of $n^{[1]}(p)$:  
$$\langle T_p \rangle = \frac{1}{2 M_p} \int_0^\Lambda dp \ p^4 \left[ n^{[1]}_{pp}(p) + n^{[1]}_{pn}(p) \right]$$

\[ \int_0^\Lambda dp \ p^2 \left[ n^{[1]}_{pp}(p) + n^{[1]}_{pn}(p) \right] \]
SRC induce inversion of kinetic energy sharing in neutron-rich nuclei

Ratio $\langle T_n = \frac{p_n^2}{2M_n} \rangle / \langle T_p = \frac{p_p^2}{2M_p} \rangle$ from computed $n^{[1]}(p)$

After correcting for SRC in LCA, minority component has largest kinetic energy (strongly depends on $N/Z$)
SRC induce inversion of kinetic energy sharing in neutron-rich nuclei

\[ \frac{\langle T_n = p_n^2 / (2M_n) \rangle}{\langle T_p = p_p^2 / (2M_p) \rangle} \text{ from computed } n^{[1]}(p) \]

After correcting for SRC in LCA, minority component has largest kinetic energy (strongly depends on \( N/Z \))

(Ghent University)
Weight of neutrons relative to protons in $n^{[1]}(p)$

$$\text{IPM: } \frac{\int_0^{p_F} dpp^2 n^{[1]}_n(p)}{\int_0^{p_F} dpp^2 n^{[1]}_p(p)}.$$  

Neutron Excess N/Z

Weight of neutrons relative to protons in $n^{[1]}(p)$

**IPM:**

$$\frac{\int_{0}^{p_F} dpp^2 n^{[1]}_n(p)}{\int_{0}^{p_F} dpp^2 n^{[1]}_p(p)}.$$  

**SRC:**

$$\frac{\int_{0.4 \text{ GeV}}^{1 \text{ GeV}} dpp^2 n^{[1]}_n(p)}{\int_{0.4 \text{ GeV}}^{1 \text{ GeV}} dpp^2 n^{[1]}_p(p)}.$$  

### Neutron Excess N/Z

- **DATA:** M. Duer et al., *Nature* **560** (2018) 617
- **Relative weight of the protons and neutrons is very different in “IPM” and “SRC” regions!**

1. **IPM:** $0.93 \frac{N}{Z} + 0.07$
2. **SRC:** $0.29 \frac{N}{Z} + 0.71$
$N/Z$ asymmetry dependence of the SRC?

Superratio of $A(e, e'N)$ for $A=\text{Al, Fe, Pb}$ relative to $C(e, e'N)$

$$\mathcal{R}_{N}^{\text{SRC/IPM}}(A) \equiv \frac{\int_{0.4 \text{ GeV}}^{1.0 \text{ GeV}} dpp^2 n_N^{[1]}(p)}{\int_{0}^{P_F} dpp^2 n_N^{[1]}(p)} \quad (N \equiv p,n)$$


(Ghent University)

INPC 2019, Glasgow
N/Z asymmetry dependence of the SRC?

Superratio of $A(e, e'N)$ for $A=\text{Al, Fe, Pb}$ relative to $C(e, e'N)$

$$R_{N}^{\text{SRC/IPM}}(A) \equiv \frac{\int_{0.4 \text{ GeV}}^{1.0 \text{ GeV}} dp^2 p^{2} n_{N}^{[1]}(p)}{\int_{0}^{p_{F}} dp^2 n_{N}^{[1]}(p)} \quad (N \equiv p, n)$$

- Weight of the minority component in the tail (SRC) part of $n^{[1]}(p)$ increases with the asymmetry $N/Z$
SUMMARY

- LCA: suited for systematic studies of SRC contributions to $n^{[1]}(p)$ and SRC-sensitive reactions
  1. Reasonable predictions for $\alpha_2$ factors
  2. $A \leq 40$: LCA predictions for fat tails in line with QMC ones
  3. Natural explanation for the “universal” behavior of the fat tails of NMD

- Distinct isospin and $N/Z$ SRC effects: in line with $A(e, e'pN)$ findings

- Neutron rich nuclei in SRC regime: protons are punching above their weight ($\approx 35\%$ in Pb)

- SRC induced spatio-temporal fluctuations in nuclei are measurable, are significant and are quantifiable
Thank you for your attention! Any questions?
Selected publications


