Higher order Symmetric Cumulants

Cindy Mordasini

Technical University of Munich

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Based on arXiv:1901.06968

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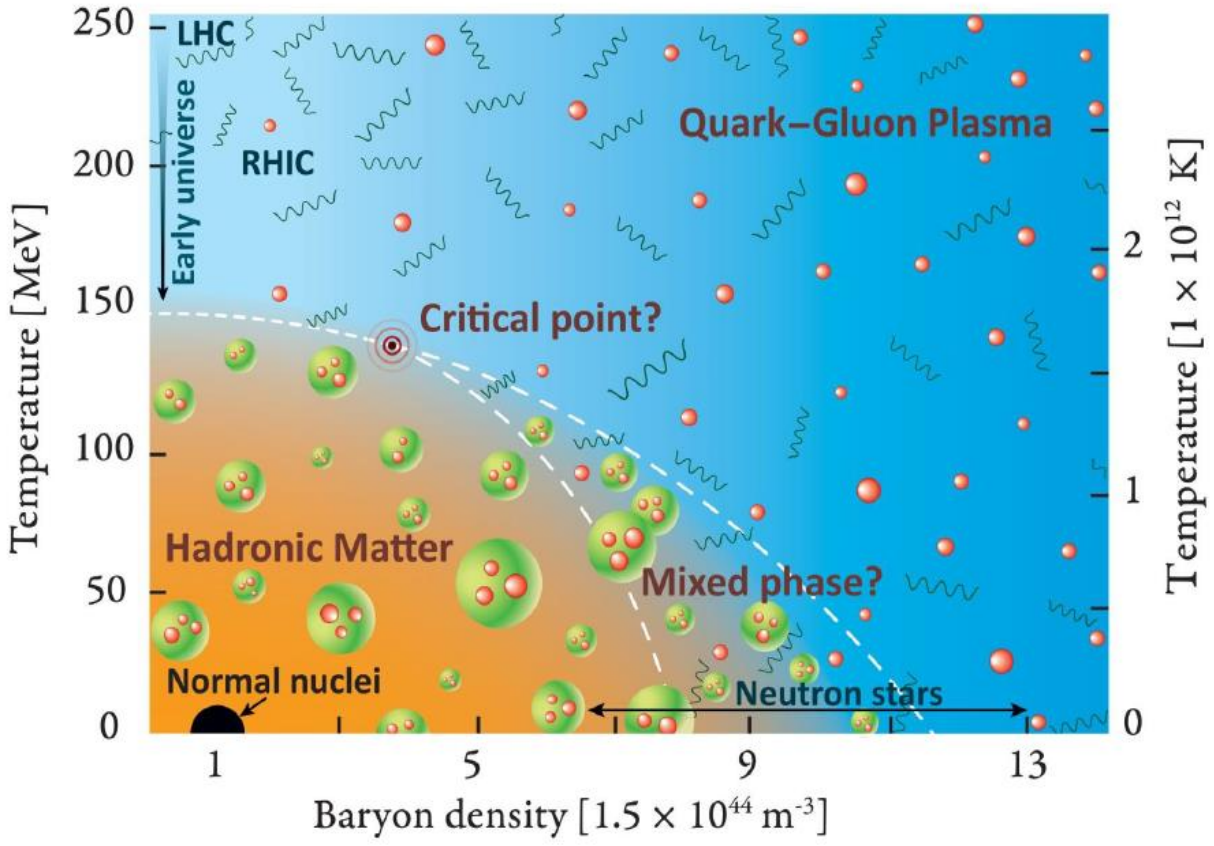
Study the Properties of Quark-Gluon Plasma

- State of matter with deconfined quarks and gluons
- Present in
  - our early Universe?
  - the core of Neutron Stars?



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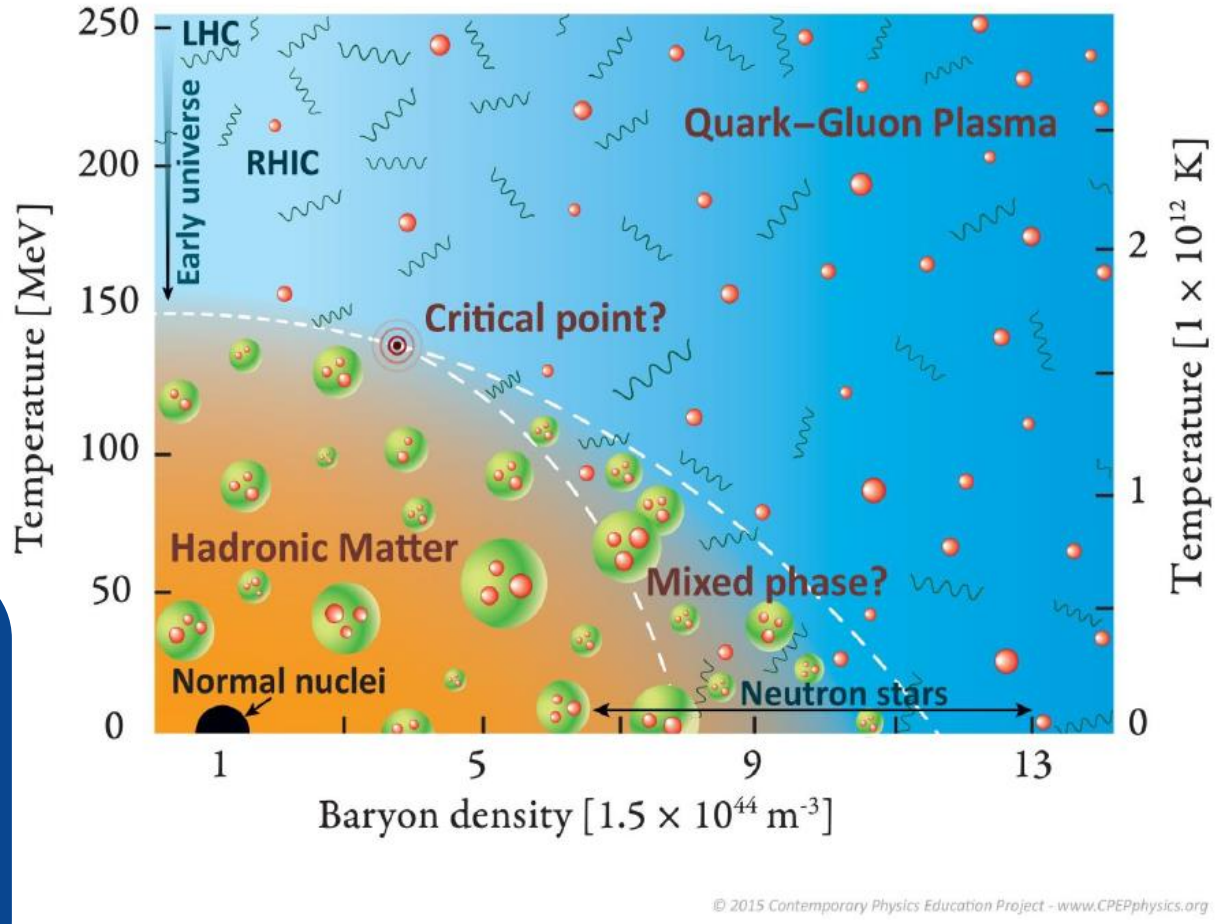
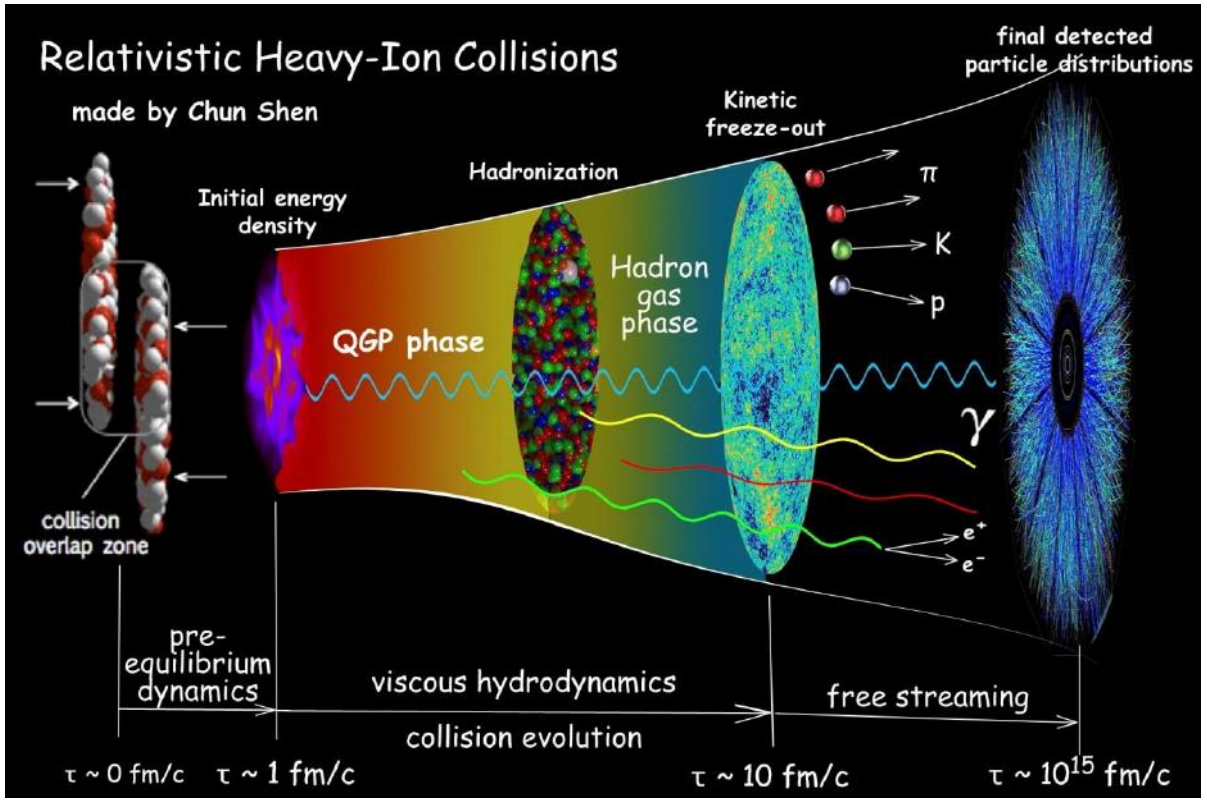


Better understanding of the Universe by studying the properties of QCD matter at extreme conditions

Create Quark-Gluon Plasma on Earth

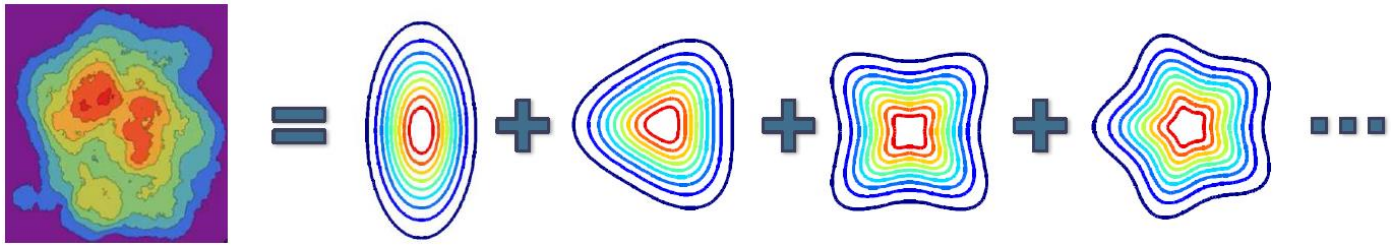
- Direct observation of QGP not possible
- Many probes to study QGP in heavy-ion collisions:
  - **Anisotropic flow**
  - Jet quenching
  - …



Study of heavy-ion collisions can provide access to the properties of the QGP

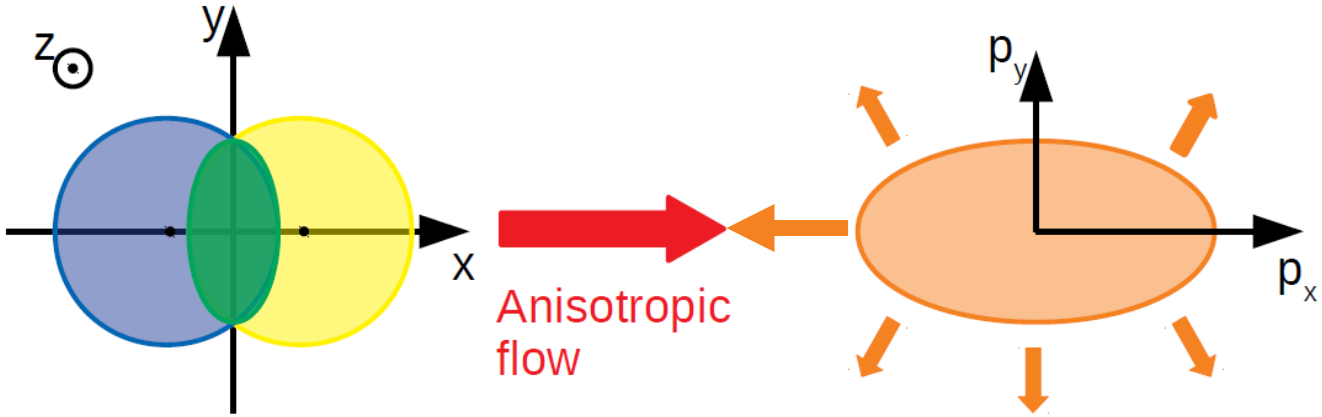
Anisotropies in the Initial and Final States

- Volume of interacting matter initially anisotropic in coordinate space



ϵ_2 ϵ_3 ϵ_4 ϵ_5

- Anisotropic flow: Transfer of the initial anisotropy into anisotropy in momentum space via the thermalized medium



Anisotropies in the Initial and Final States

- Described by Fourier series:

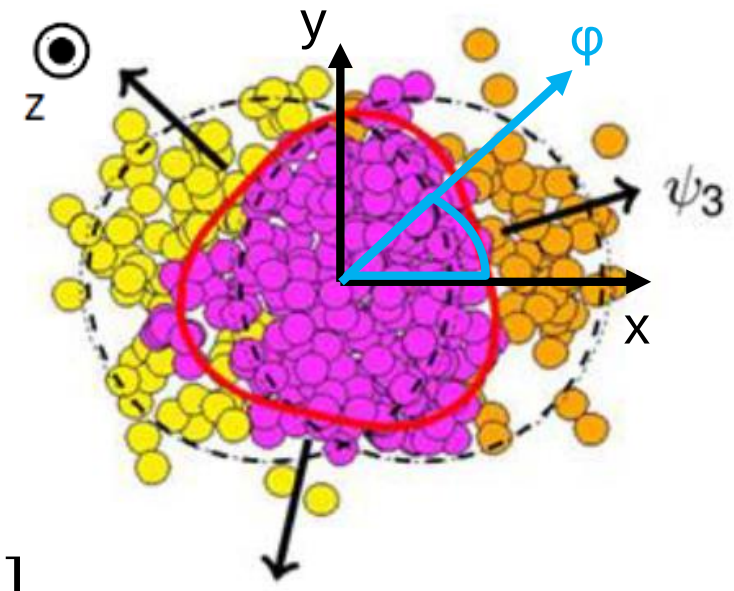
$$f(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]\right]$$

S. Voloshin and Y. Zhang, Z. Phys. C **70** (1996) 665

- v_n and Ψ_n related to φ:

$$\langle\cos[n_1\varphi_1 + \cdots + n_k\varphi_k]\rangle = v_{n_1} \ldots v_{n_k} \cos\left[n_1\Psi_{n_1} + \cdots + n_k\Psi_{n_k}\right]$$

R.S. Bhalerao, M. Luzum and J.-Y. Ollitrault, PRC 84, 034910 (2011)



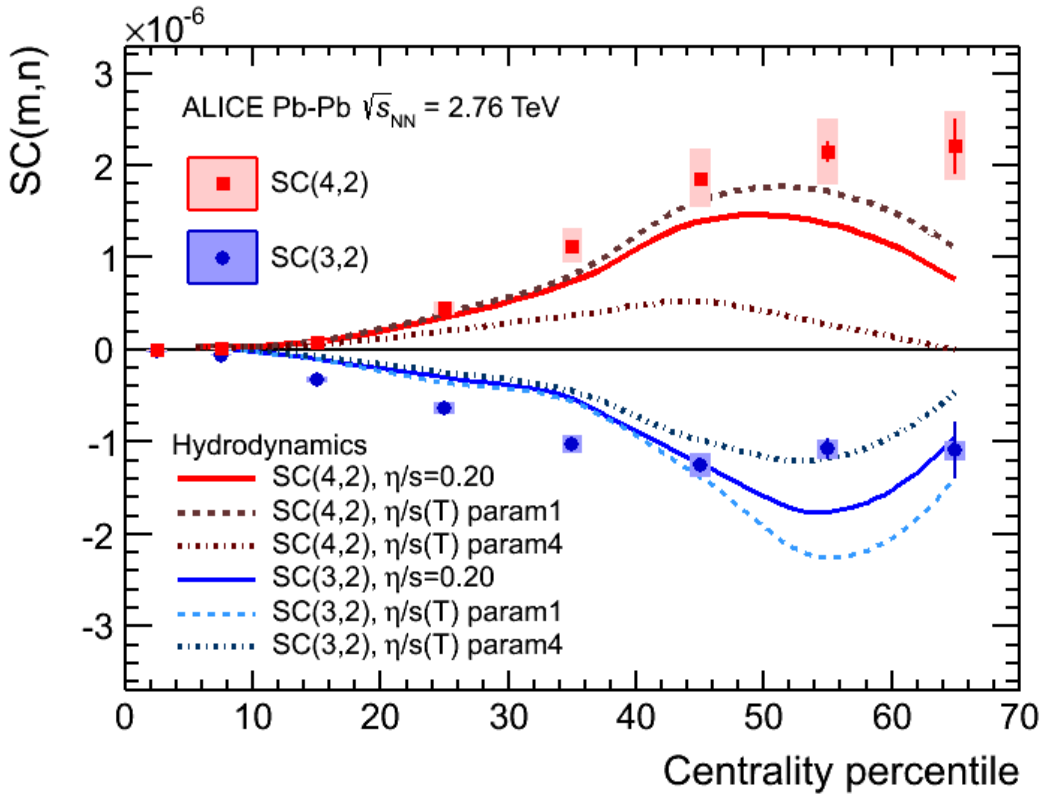
φ: Azimuthal angle
v_n: Flow harmonic
Ψ_n: Symmetry-plane

Flow and Properties of Quark-Gluon Plasma

- Measurements of v_n sensitive to properties of QGP
- Correlations between two harmonics probed by:

$$\mathrm{SC}(m,n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

- New and independent set of constraints on the system



ALICE Collaboration, PRL 117, 182301 (2016)

Flow and Properties of Quark-Gluon Plasma

- Combinations of v_n and Ψ_n sensitive to properties of QGP
- Correlations between two harmonics probed by:

$$\mathrm{SC}(m,n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

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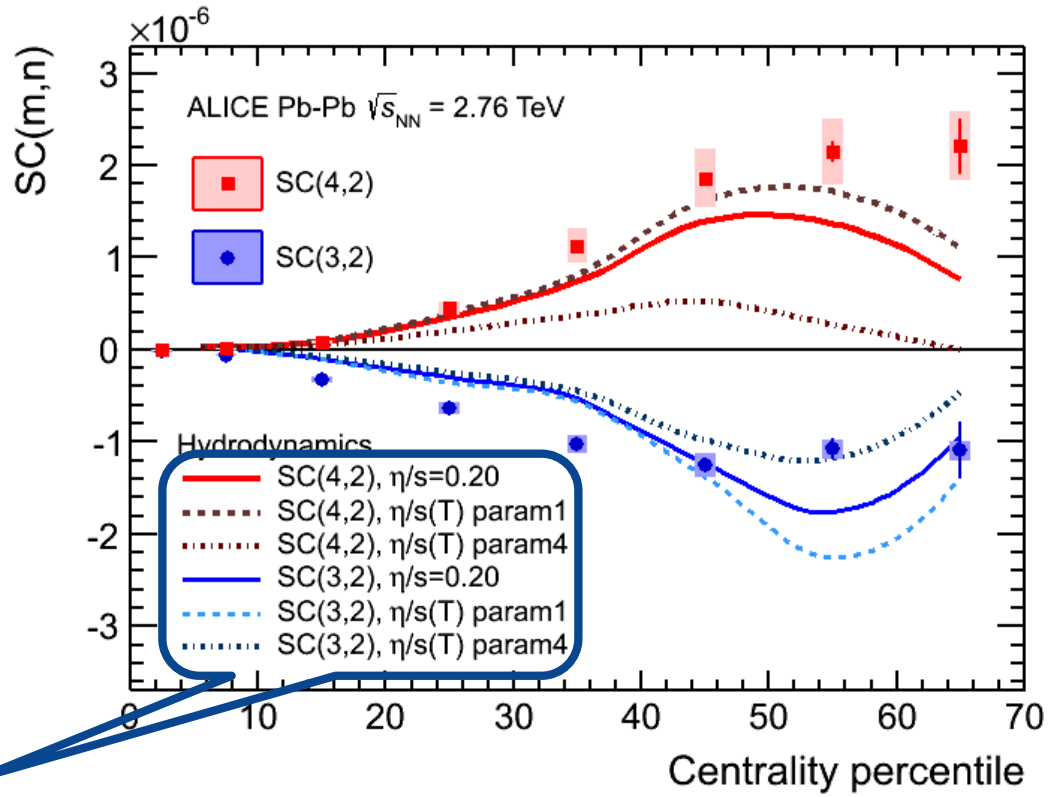


ALICE Collaboration, PRL 117, 182301 (2016)

Sensitivity to η/s of QGP (not accessible with only one harmonic)

Generalisation to three Different Harmonics

- Are there genuine correlations between more than two harmonics?

  → Possible new constraints for the system

  (Not accessible with $\mathrm{SC}(m,n)$ or only one harmonic)

"Higher order Symmetric Cumulants"

CM, A. Bilandzic, D. Karakoç and S. F. Taghavi, arXiv:1901.06968

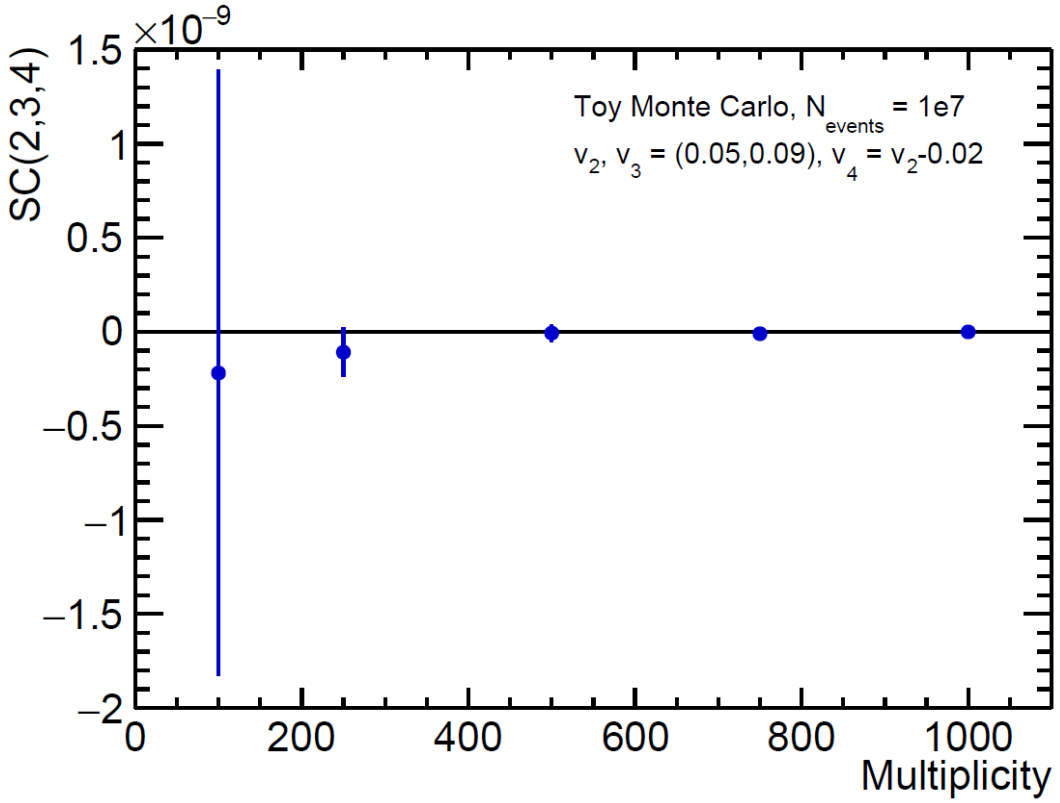
- Example: 3-harmonic Symmetric Cumulants:

$$\mathrm{SC}(k,l,m) = \langle v_k^2 v_l^2 v_m^2 \rangle - \langle v_k^2 v_l^2 \rangle \langle v_m^2 \rangle - \langle v_k^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_l^2 v_m^2 \rangle \langle v_k^2 \rangle + 2 \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle$$

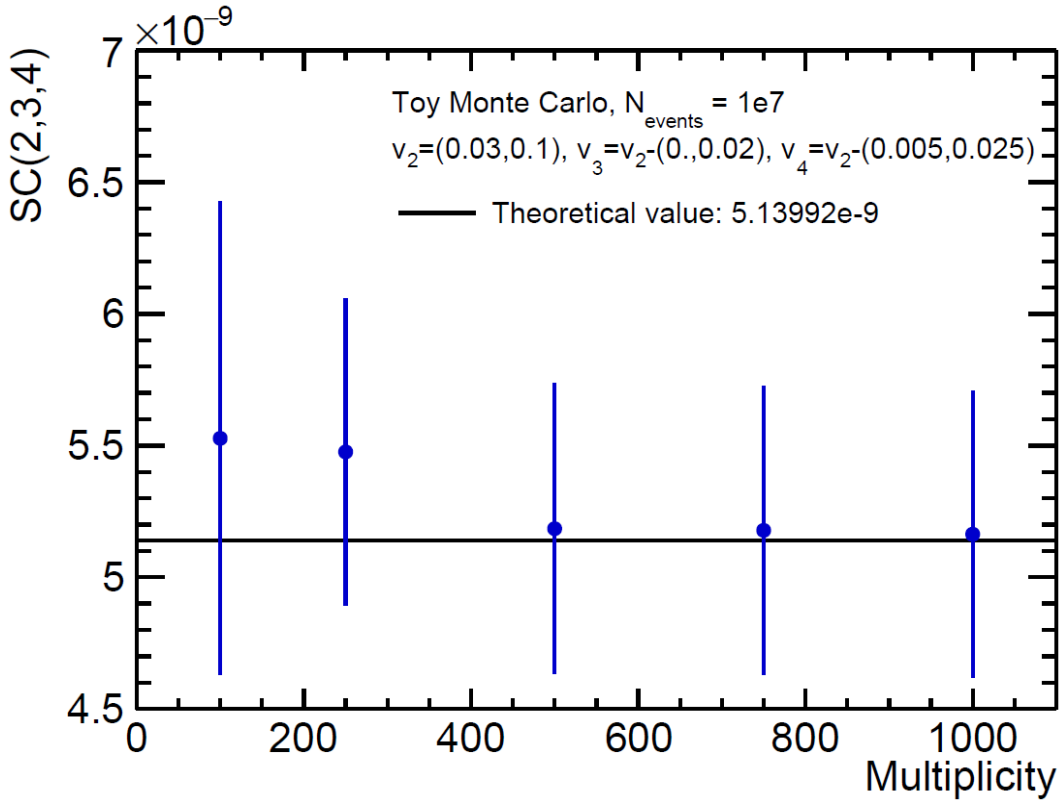
$$\mathrm{SC}_\epsilon(k,l,m) = \langle \epsilon_k^2 \epsilon_l^2 \epsilon_m^2 \rangle - \langle \epsilon_k^2 \epsilon_l^2 \rangle \langle \epsilon_m^2 \rangle - \langle \epsilon_k^2 \epsilon_m^2 \rangle \langle \epsilon_l^2 \rangle - \langle \epsilon_l^2 \epsilon_m^2 \rangle \langle \epsilon_k^2 \rangle + 2 \langle \epsilon_k^2 \rangle \langle \epsilon_l^2 \rangle \langle \epsilon_m^2 \rangle$$

Validation with Toy Monte Carlo Simulations

- Less than 3 correlated harmonics → SC(2,3,4) = 0



- 3 correlated harmonics → SC(2,3,4) ≠ 0



From Toy Simulations to Realistic Models

- Toy Monte Carlo simulations good verification tool…

… But what are the predictions for more realistic models?

From Toy Simulations to Realistic Models

- Toy Monte Carlo simulations good verification tool…

… But what are the predictions for more realistic models?

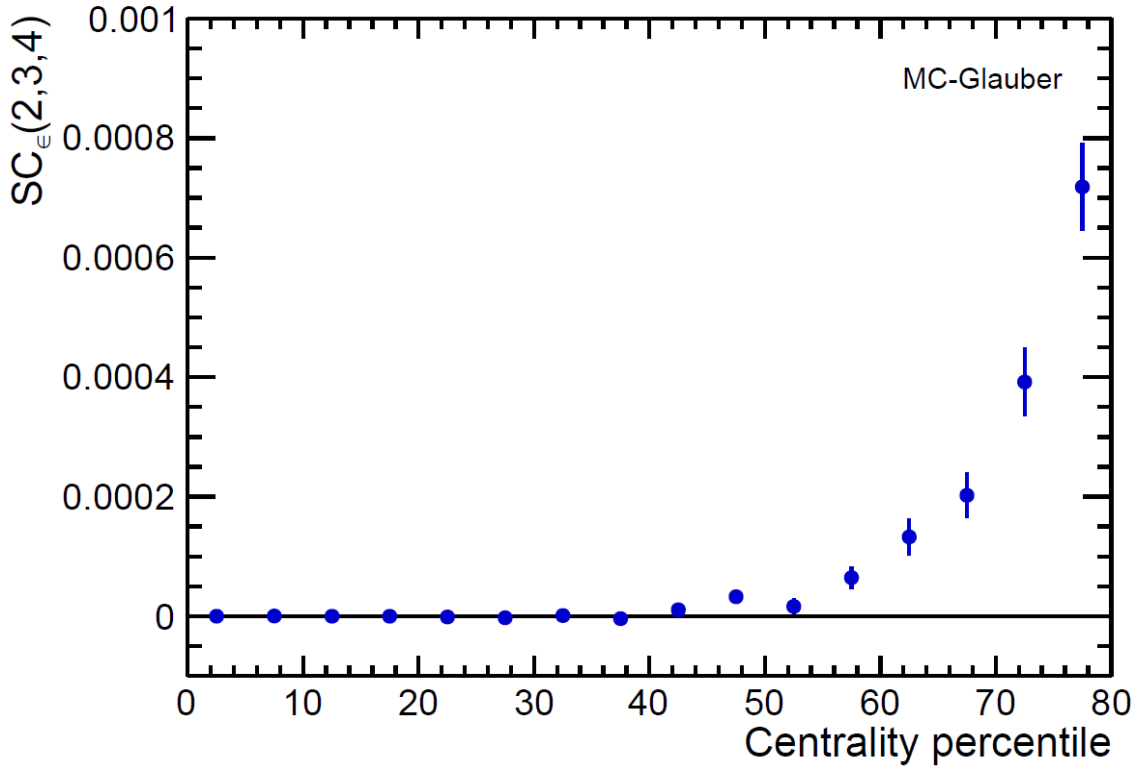
- Predictions from iEBE-VISHNU
  - Event generator for heavy-ion collisions
  - Describes the evolution of the collision
  - Simulations → Access to $\mathrm{SC}_{\epsilon}(k,l,m)$ and $\mathrm{SC}(k,l,m)$
  - 14000 simulated events per centrality bin → Feasibility test

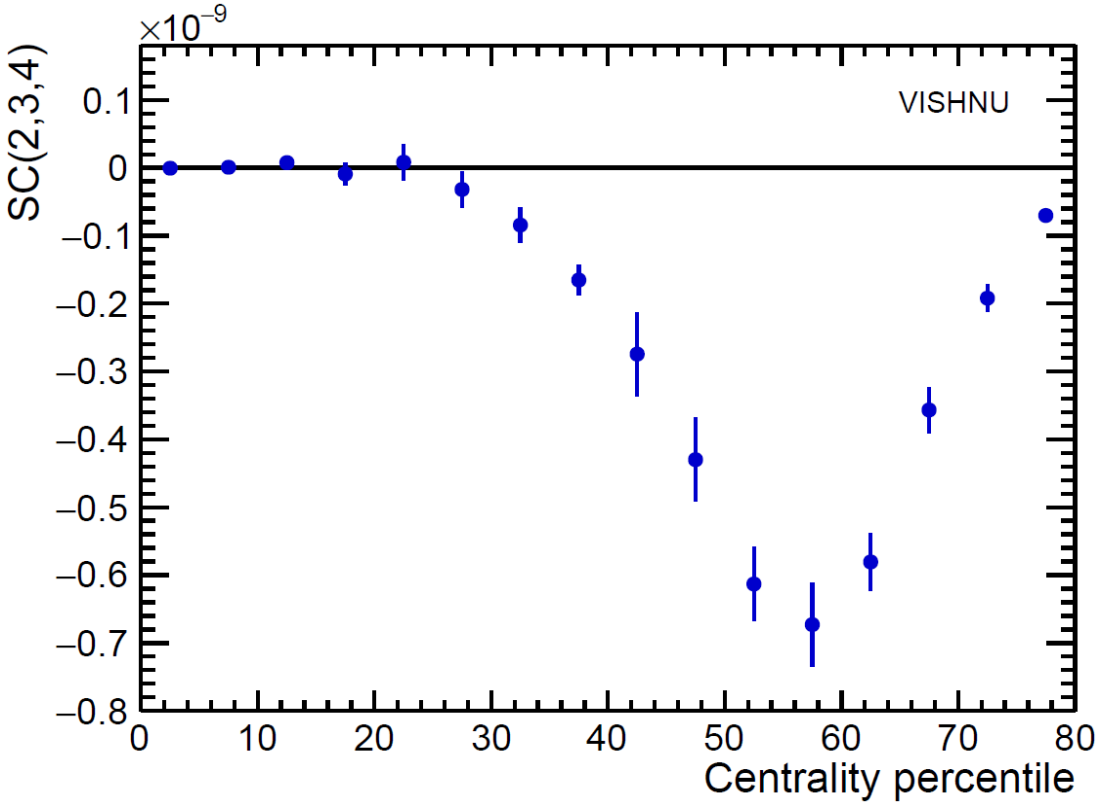
Initial state from MC-Glauber model

After 0.6 fm/c, flow from the hydrodynamic evolution with η/s = 0.08

Predictions with iEBE-VISHNU

- System smaller for peripheral collisions → Harder to transfer the initial anisotropy to the final state





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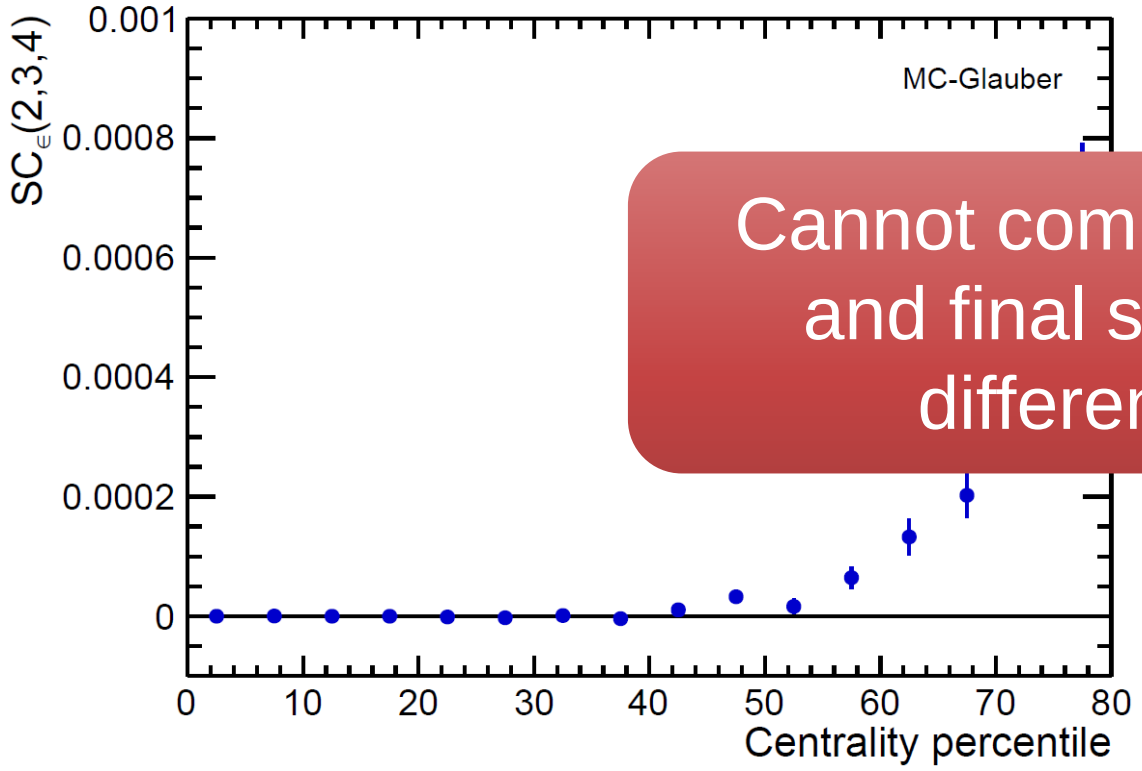
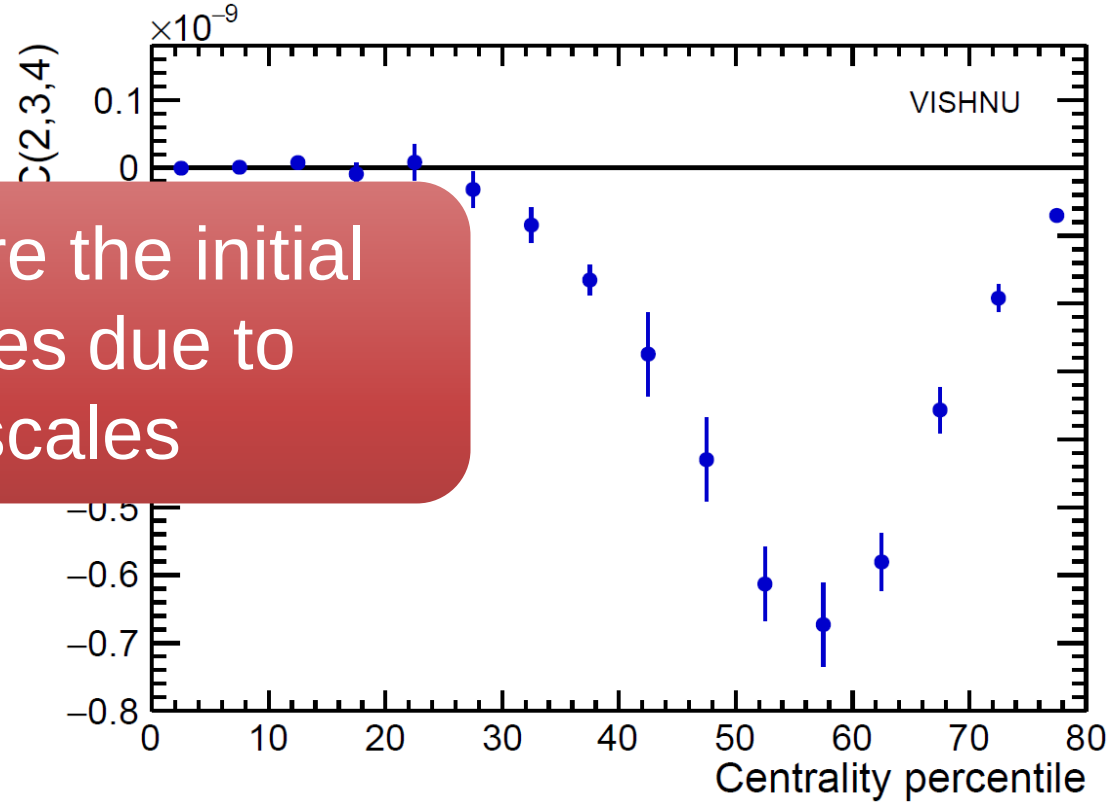
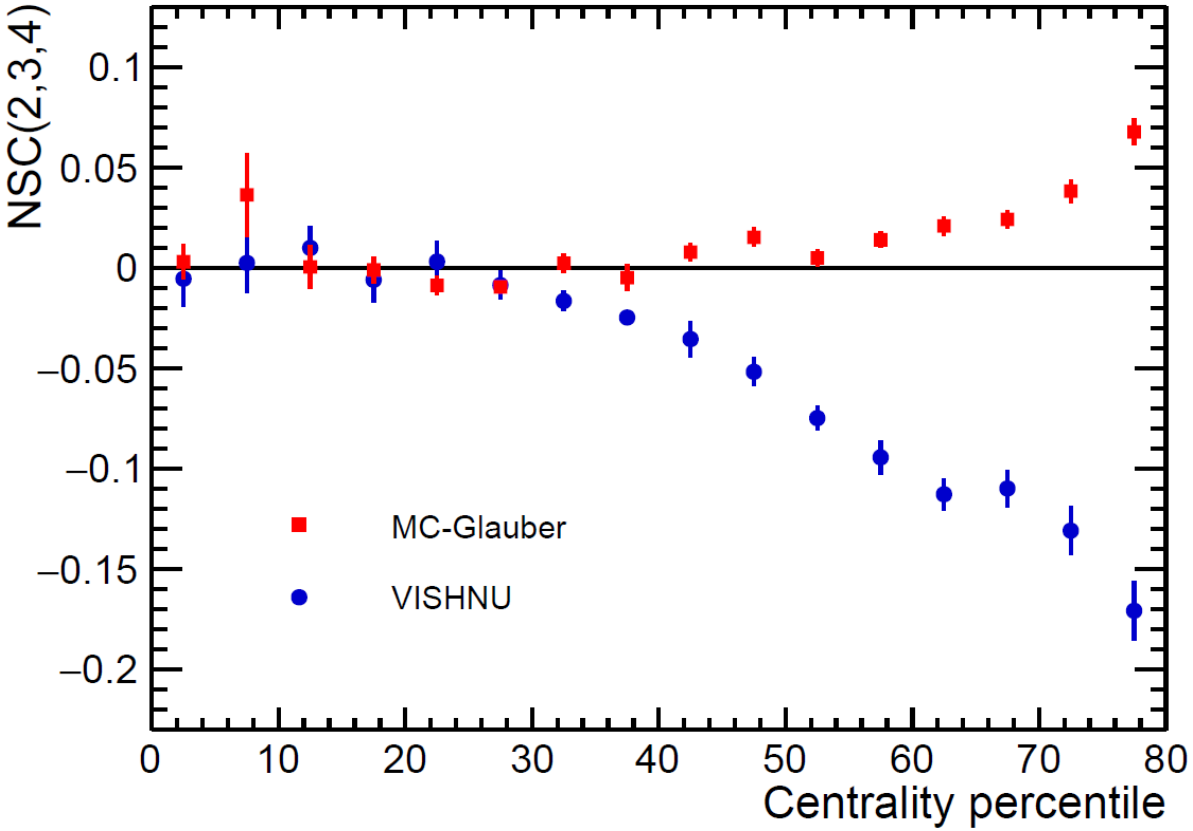




Cannot compare the initial and final states due to different scales

Normalised Symmetric Cumulants

- $\mathrm{NSC}(k,l,m) = \frac{\mathrm{SC}(k,l,m)}{\langle v_k^2\rangle\langle v_l^2\rangle\langle v_m^2\rangle}$ and $\mathrm{NSC}_\epsilon(k,l,m) = \frac{\mathrm{SC}_\epsilon(k,l,m)}{\langle \epsilon_k^2\rangle\langle \epsilon_l^2\rangle\langle \epsilon_m^2\rangle}$
- How much of the final particle correlations come from the initial state?
- Sign change between $\mathrm{NSC}(2,3,4)$ and $\mathrm{NSC}_\epsilon(2,3,4)$
  - SC(4,2) > 0
  - SC(3,2) < 0



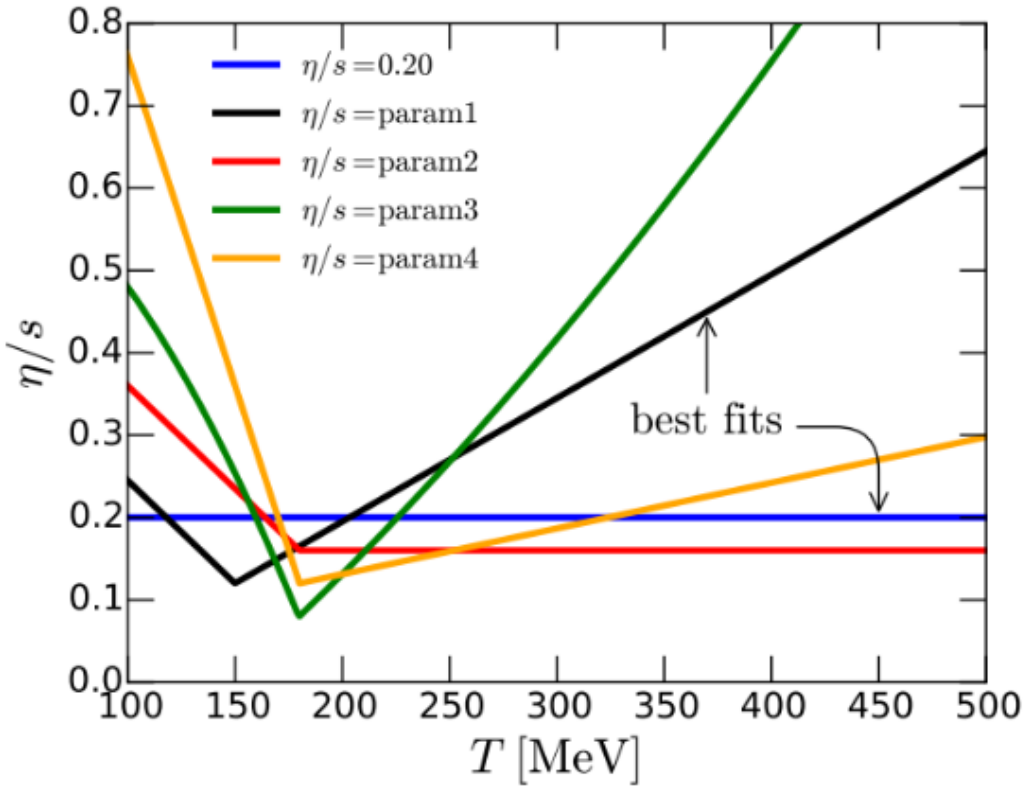
Summary and Outlook

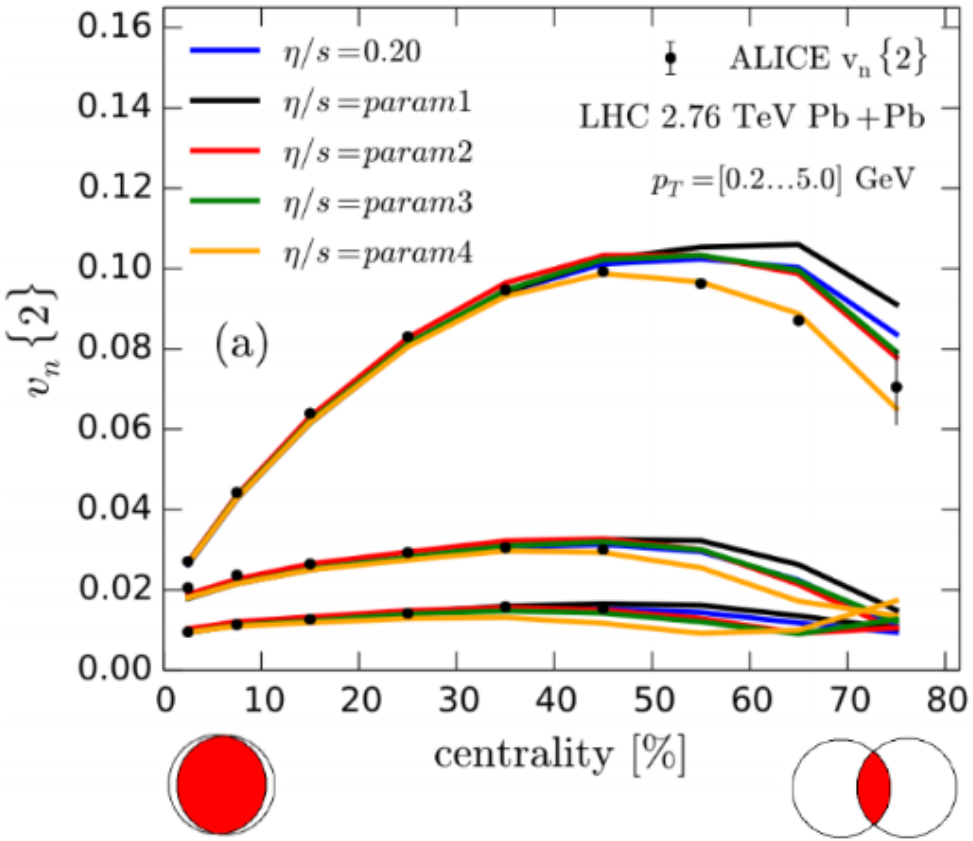
- Introduced the generalisation to higher order Symmetric Cumulants
- Validated SC(k,l,m) with Toy Monte Carlo simulations
- Showed predictions from iEBE-VISHNU

- **Next step?** Analyses of SC(k,l,m) on ALICE Pb-Pb data from RUN 1 and RUN 2

Backups

Parametrisations for η/s





H. Niemi, K. J. Eskola, R. Paatelainen, Phys. Rev. C 93, 024907 (2016)

Cumulants

- Cumulant $\langle \ldots \rangle_c$ describes the genuine correlations between all the elements in the correlator
  - Term unique in the decomposition
- Cumulants not directly measurable → Must be computed from the correlations
- 2-particle cumulant: $\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$
- ➔ In 3-particle cumulant:

$$\langle X_1 X_2 X_3 \rangle_c = \langle X_1 X_2 X_3 \rangle - \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle + 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle$$

  - Recursive method for higher order cumulants
- Study correlations
  - Different k-particle correlators: $X_j = e^{in\varphi_j}$ (N. Borghini, P. M. Dinh and J.-Y. Ollitrault, PRC 63, 054906 (2001))
  - Different harmonics: e.g. Symmetric Cumulants

Cumulants

- Cumulants not directly measurable ⇒ Must be computed from the correlations
- 2-particle cumulant: $\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$
- In 3-particle cumulant:

$$\langle X_1 X_2 X_3 \rangle_c = \langle X_1 X_2 X_3 \rangle - \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle + 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle$$

- Recursive method for higher order cumulants

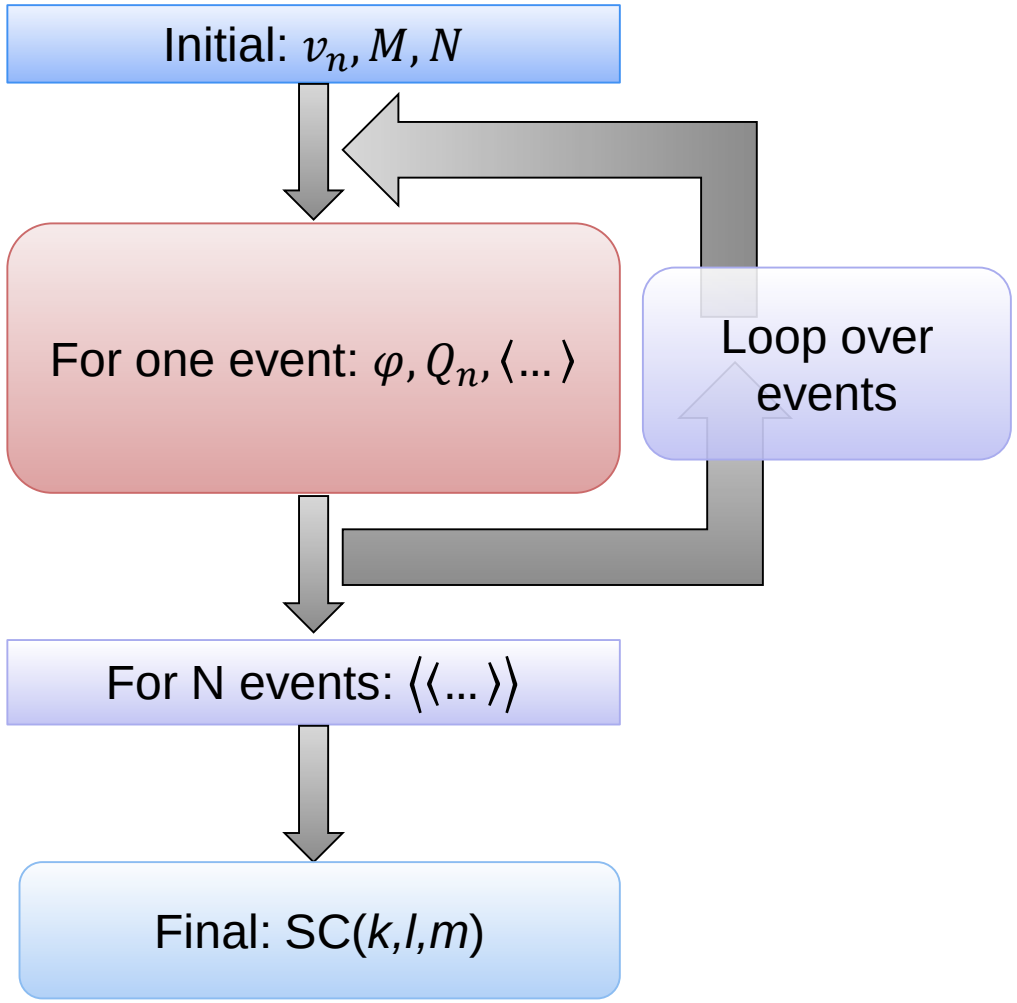
Requirements to be a Symmetric Cumulant

1) No built-in trivial contribution: if v_k constant → SC(k,l,m) = 0
2) Genuine 3-harmonic correlations: SC(k,l,m) ≠ 0 only for 3 correlated harmonics
3) Symmetry: SC(k,l,m) = SC(l,m,k) = SC(k,m,l) = …
4) Cleanliness: no dependence on symmetry planes in SC by definition
5) Isotropy: all correlators used in SC must be isotropic
6) Uniqueness: no ambiguity in the combination of correlators used to get the final expression
7) Robustness against nonflow: if only nonflow in the system → expected scaling of SC(k,l,m)
8) Multiplicity weights: which weights to use to go from $\langle \ldots \rangle$ to $\langle\langle \ldots \rangle\rangle$?

→ **How to check if SC(*k*,*l*,*m*) follows the requirements? Toy Monte Carlo simulations**

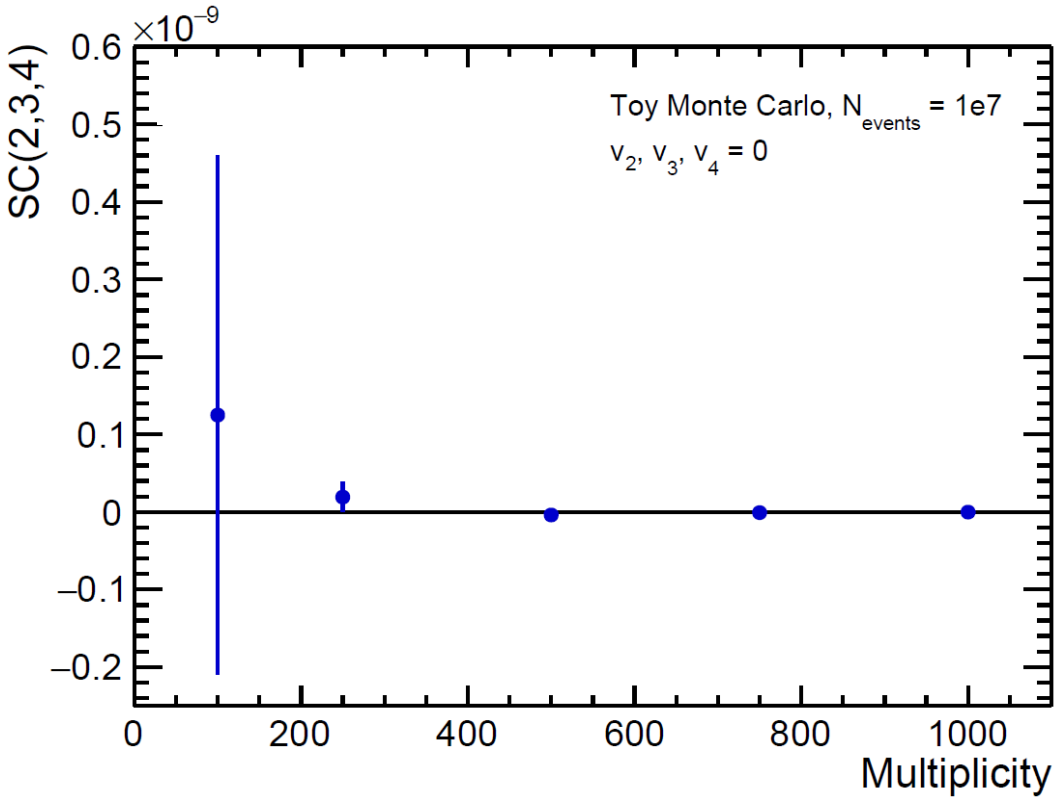
Toy Monte Carlo Setup

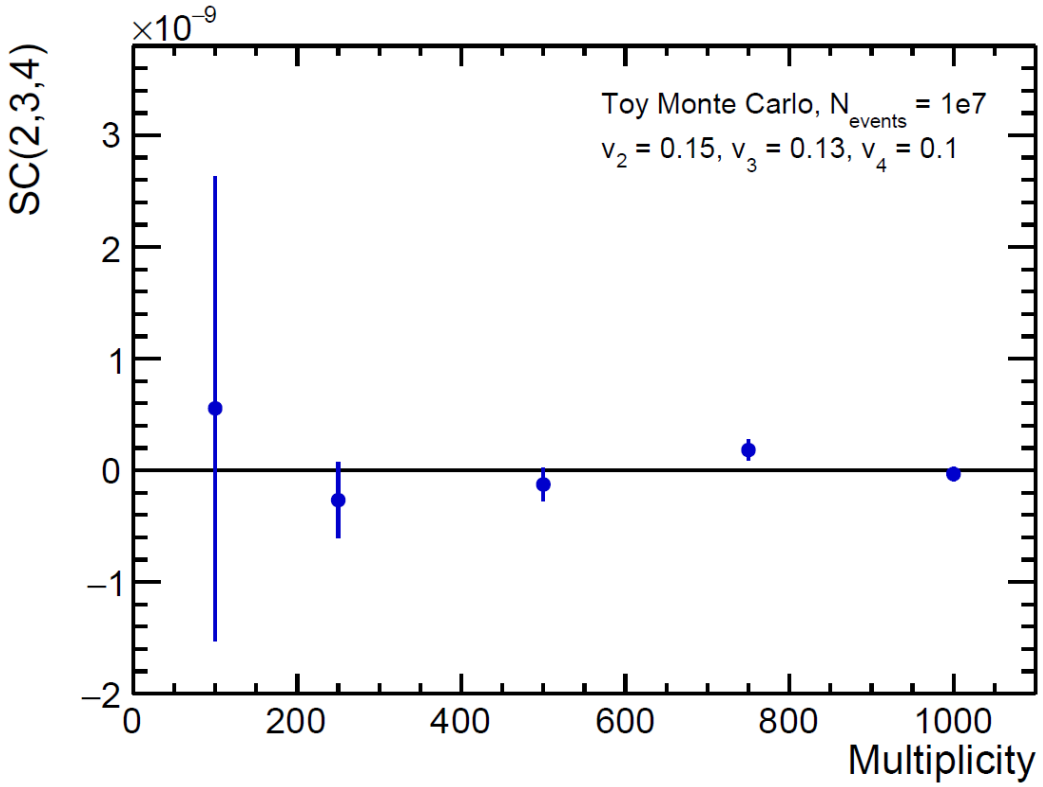
- Controlled environment to test the influence of the initial parameters
- Setup implemented for this study:
  - Set the flow harmonics v_n in the Fourier series
  - Uniform sampling of azimuthal angles φ for M particles
  - Compute the Q-vectors Q_n for the event
  - Compute the needed correlators $\langle \ldots \rangle$ from the Q-vectors for the event
  - Loop over N events to compute $\langle\langle \ldots \rangle\rangle$
  - Compute SC(k,l,m)



1) No built-in Trivial Contribution?

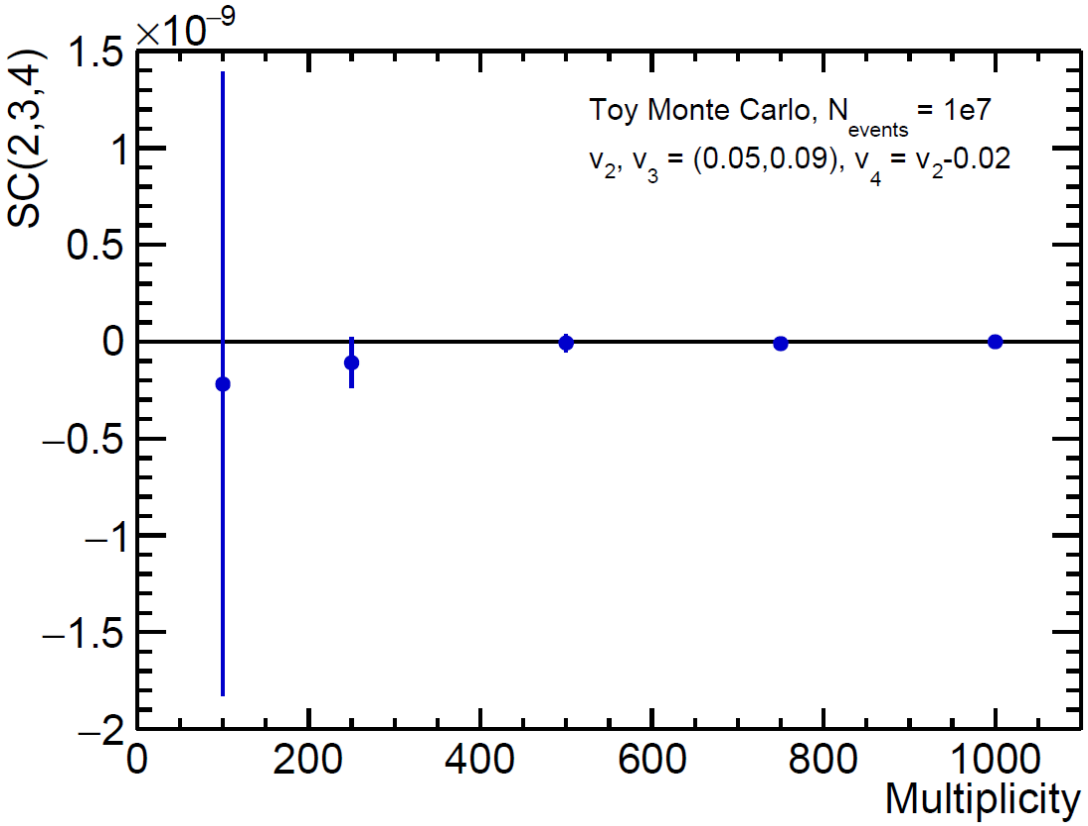
- v_2, v_3 and v_4 constant ➔ SC(2,3,4) = 0
- No built-in contribution

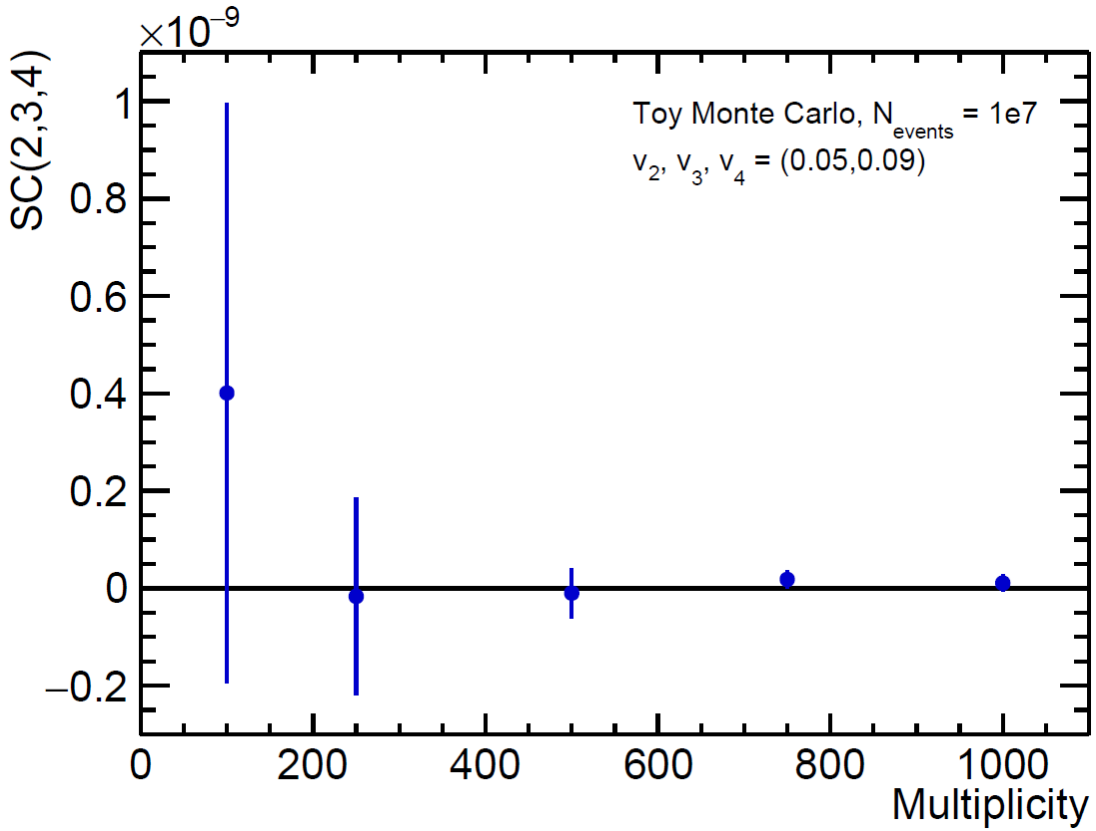




2) Genuine 3-harmonic Correlations?

- SC(2,3,4) compatible with 0 for all multiplicities



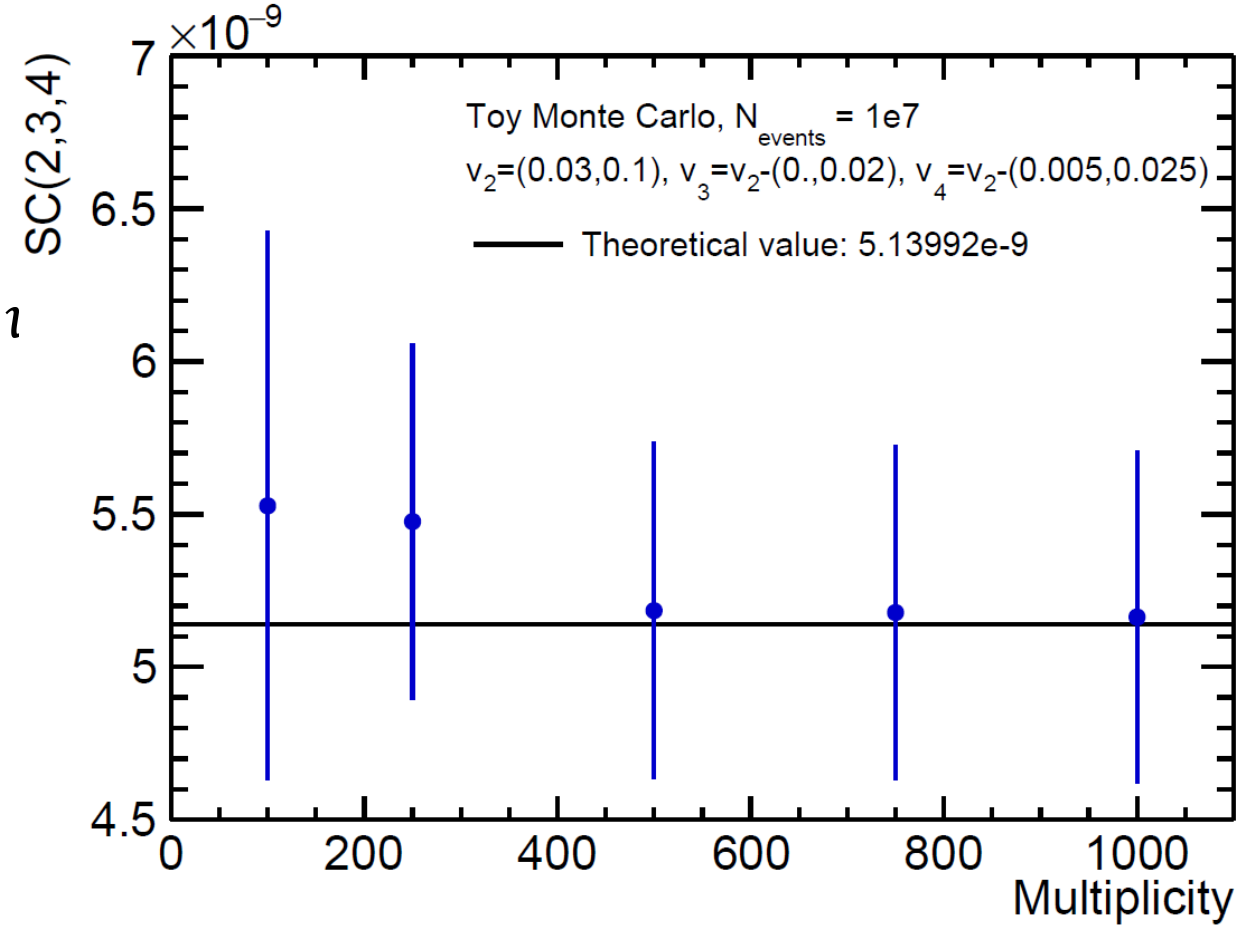


➔ SC(*k*,*l*,*m*) not sensitive to less than 3 correlated harmonics

2) Genuine 3-harmonic Correlations?

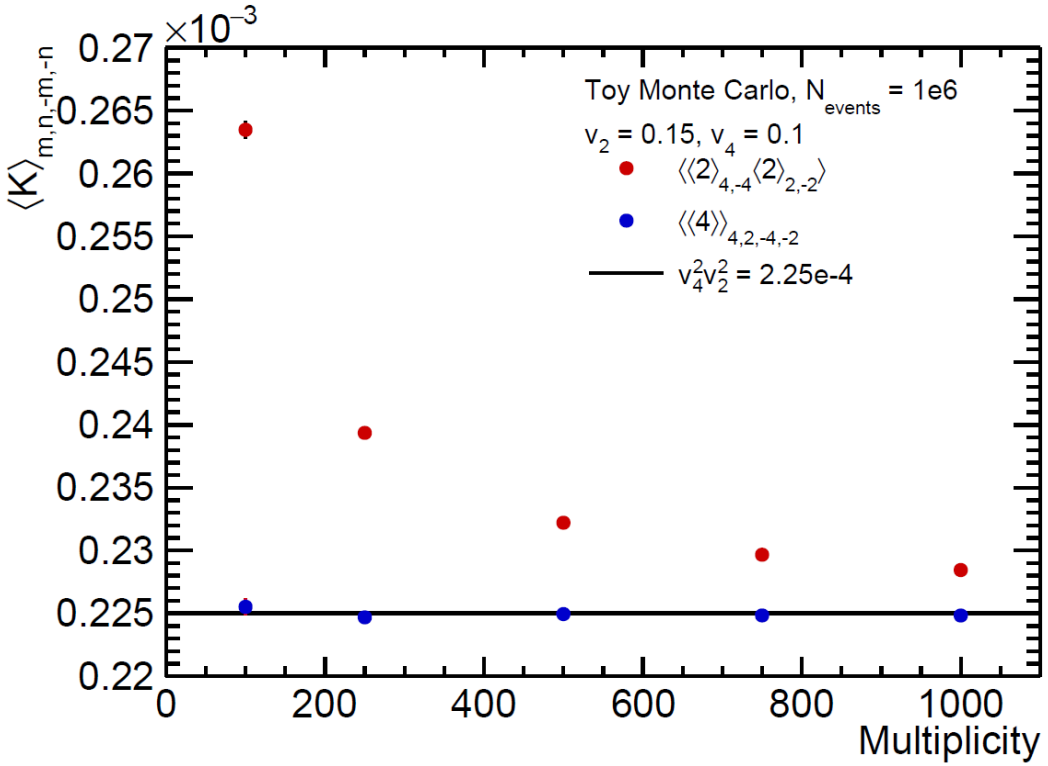
- Simple model of correlations among 3 harmonics: $f(v_2, v_3, v_4)$ in: $< v_2^a v_3^b v_4^c > = \iiint v_2^a v_3^b v_4^c f(v_2, v_3, v_4) dv_2 dv_3 d$
- Compute all terms of SC(2,3,4)
- Simulations in agreement with theory for all multiplicities

➔ Sensitive to 3-harmonic correlations only



6) Uniqueness in SC(*m*,*n*)?

- Ambiguity in $\langle v_m^2 v_n^2 \rangle$ from $\langle \cos[n_1\varphi_1 + \cdots + n_k\varphi_k] \rangle = v_{n_1} \ldots v_{n_k} \cos[n_1\Psi_{n_1} + \cdots + n_k\Psi_{n_k}]$
- SC(*m*,*n*) can be written as
  - Usual: $\langle\langle \cos(m(\varphi_1 - \varphi_2) + n(\varphi_3 - \varphi_4)) \rangle\rangle$
    $- \langle\langle \cos(m(\varphi_1 - \varphi_2)) \rangle\rangle \langle\langle \cos(n(\varphi_1 - \varphi_2)) \rangle\rangle$
  - Alternative:
    $\langle\langle \cos(m(\varphi_1 - \varphi_2)) \rangle \langle \cos(n(\varphi_1 - \varphi_2)) \rangle\rangle$
    $- \langle\langle \cos(m(\varphi_1 - \varphi_2)) \rangle\rangle \langle\langle \cos(n(\varphi_1 - \varphi_2)) \rangle\rangle$
- From mathematical derivation:
  - $\langle\langle \cos(m(\varphi_1 - \varphi_2) + n(\varphi_3 - \varphi_4)) \rangle\rangle$ works
  - Autocorrelations in $\langle\langle \cos(m(\varphi_1 - \varphi_2)) \rangle \langle \cos(n(\varphi_1 - \varphi_2)) \rangle\rangle$



6) Uniqueness?

- As for SC(*m*,*n*), many expressions lead in theory to the same final expression for SC(*k*,*l*,*m*)
  - Again from $\langle\cos[n_1\varphi_1 + \cdots + n_k\varphi_k]\rangle = v_{n_1} \ldots v_{n_k} \cos\left[n_1\Psi_{n_1} + \cdots + n_k\Psi_{n_k}\right]$
- Example for the first term: $\left\langle v_k^2 v_l^2 v_m^2 \right\rangle$

  → $\left\langle\langle\cos(k(\varphi_1 - \varphi_2) + l(\varphi_3 - \varphi_4) + m(\varphi_5 - \varphi_6))\rangle\right\rangle$?

  → $\left\langle\langle\cos(k(\varphi_1 - \varphi_2))\rangle\langle\cos(l(\varphi_1 - \varphi_2))\rangle\langle\cos(m(\varphi_1 - \varphi_2))\rangle\right\rangle$?

  → $\left\langle\langle\cos(k(\varphi_1 - \varphi_2) + l(\varphi_3 - \varphi_4))\rangle\langle\cos(m(\varphi_1 - \varphi_2))\rangle\right\rangle$?
- Mathematical derivation too tedious for 3 different harmonics
  - Use of the Toy Monte Carlo setup for 3 different expressions

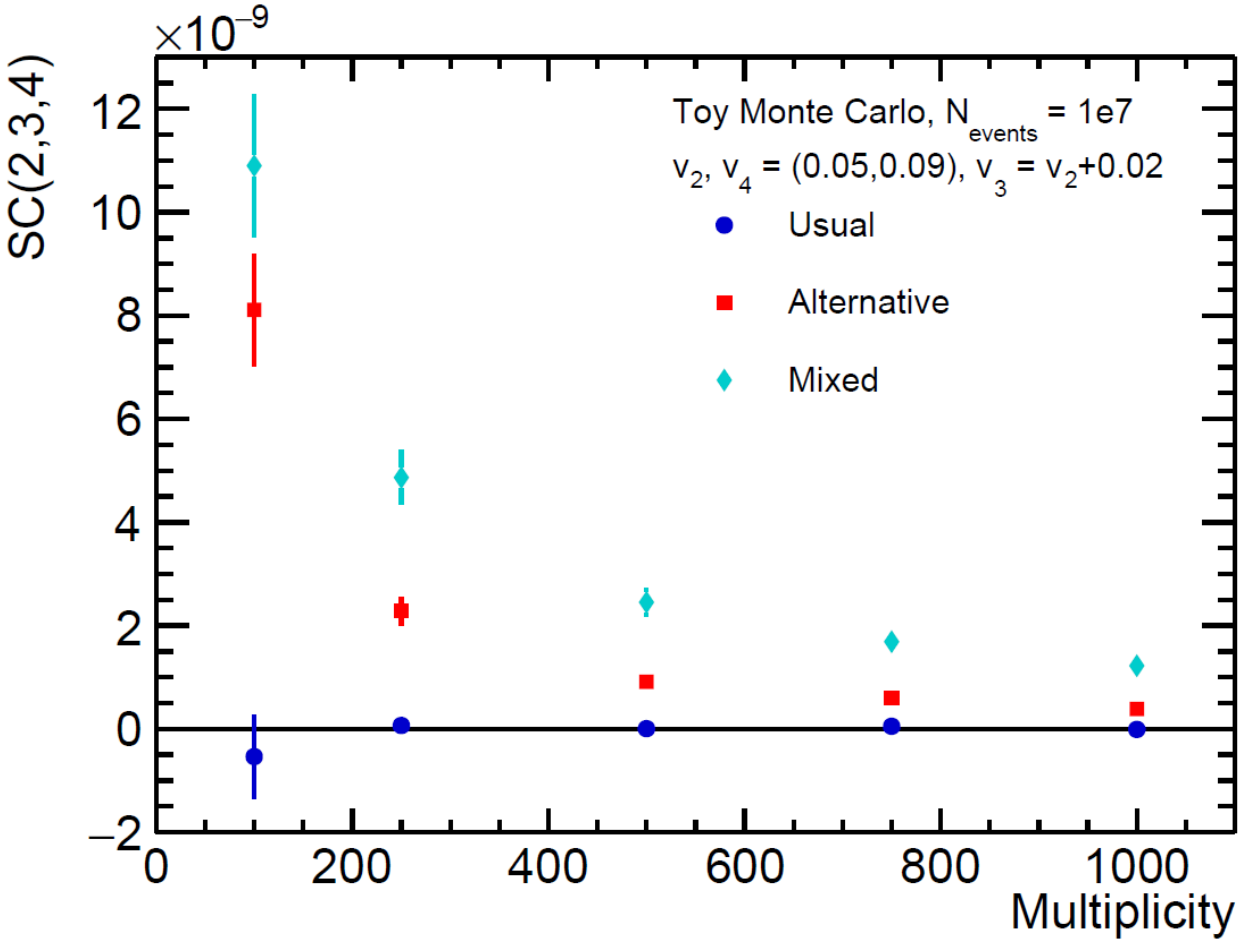
6) Uniqueness?

- Usual: $SC(k,l,m) = \langle\langle 6\rangle\rangle_{k,l,m,-k,-l,-m} - \langle\langle 4\rangle\rangle_{k,l,-k,-l}\langle\langle 2\rangle\rangle_{m,-m} - \langle\langle 4\rangle\rangle_{k,m,-k,-m}\langle\langle 2\rangle\rangle_{l,-l} - \langle\langle 4\rangle\rangle_{l,m,-l,-m}\langle\langle 2\rangle\rangle_{k,-k} + 2\,\langle\langle 2\rangle\rangle_{k,-k}\langle\langle 2\rangle\rangle_{l,-l}\langle\langle 2\rangle\rangle_{m,-m}$
- Alternative: $SC(k,l,m) = \left\langle\langle 2\rangle_{k,-k}\langle 2\rangle_{l,-l}\langle 2\rangle_{m,-m}\right\rangle - \left\langle\langle 2\rangle_{k,-k}\langle 2\rangle_{l,-l}\right\rangle\langle\langle 2\rangle\rangle_{m,-m} - \left\langle\langle 2\rangle_{k,-k}\langle 2\rangle_{m,-m}\right\rangle\langle\langle 2\rangle\rangle_{l,-l} - \left\langle\langle 2\rangle_{l,-l}\langle 2\rangle_{m,-m}\right\rangle\langle\langle 2\rangle\rangle_{k,-k} + 2\,\langle\langle 2\rangle\rangle_{k,-k}\langle\langle 2\rangle\rangle_{l,-l}\langle\langle 2\rangle\rangle_{m,-m}$
- Mixed: $SC(k,l,m) = \left\langle\langle 4\rangle_{k,-l,-k,-l}\langle 2\rangle_{m,-m}\right\rangle - \left\langle\langle 2\rangle_{k,-k}\langle 2\rangle_{l,-l}\right\rangle\langle\langle 2\rangle\rangle_{m,-m} - \left\langle\langle 2\rangle_{k,-k}\langle 2\rangle_{m,-m}\right\rangle\langle\langle 2\rangle\rangle_{l,-l} - \left\langle\langle 2\rangle_{l,-l}\langle 2\rangle_{m,-m}\right\rangle\langle\langle 2\rangle\rangle_{k,-k} + 2\,\langle\langle 2\rangle\rangle_{k,-k}\langle\langle 2\rangle\rangle_{l,-l}\langle\langle 2\rangle\rangle_{m,-m}$

6) Uniqueness?

- Simulation with two anticorrelated harmonics
  - Expected: zero for all multiplic
- Usual: ok
- Alternative and mixed:
  - Multiplicity dependence
  - Presence of autocorrelations

➔ Unique way to write SC(*k,l,n*
terms of azimuthal angles

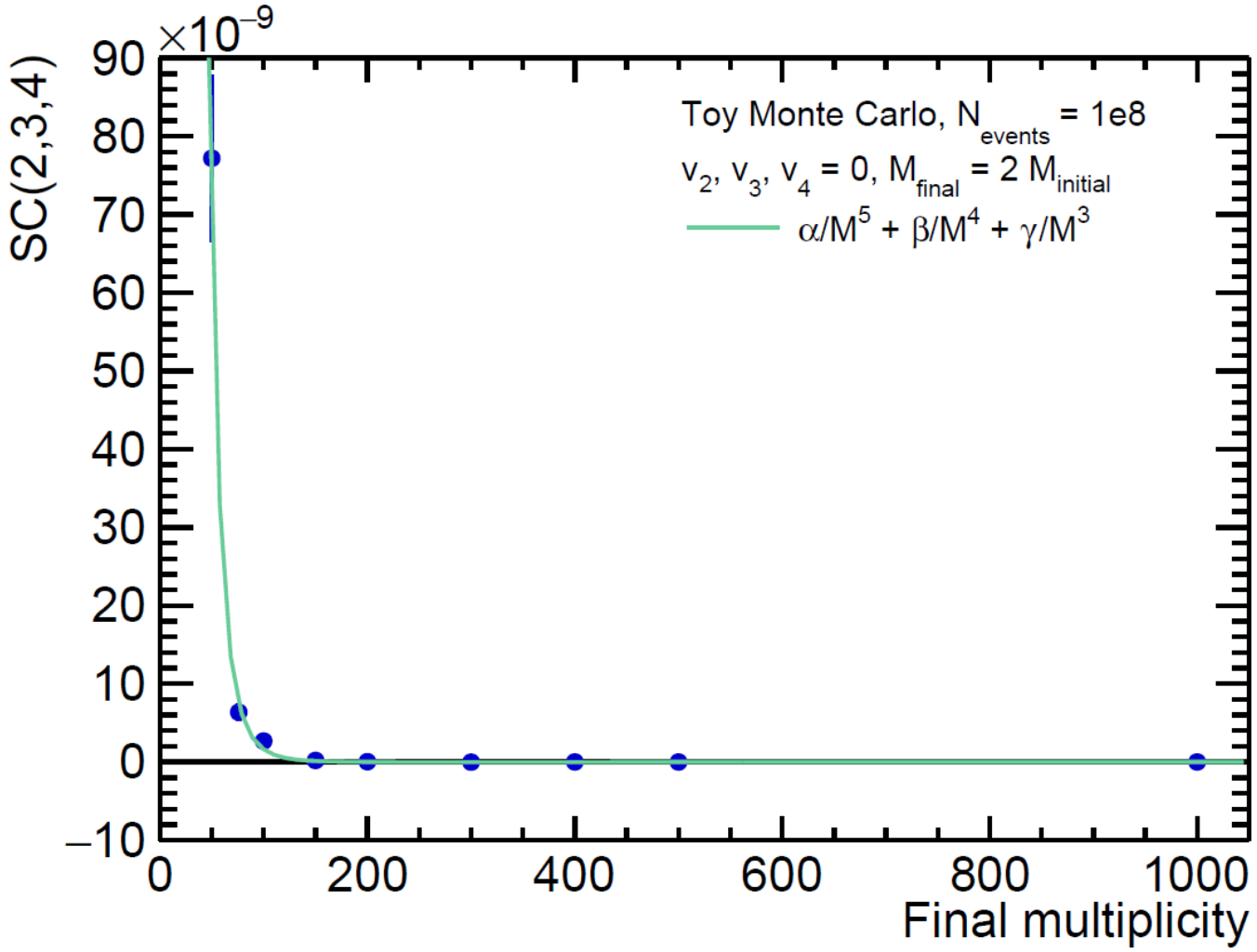


7) Robustness against Nonflow?

- $SC(k,l,m) = \langle v_k^2 v_l^2 v_m^2 \rangle - \langle v_k^2 v_l^2 \rangle \langle v_m^2 \rangle - \langle v_k^2 v_m^2 \rangle \langle v_l^2 \rangle - \langle v_l^2 v_m^2 \rangle \langle v_k^2 \rangle + 2 \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle$
- Expected behavior: $\delta_{SC(k,l,m)} \sim \frac{\alpha}{M^{6-1}} + \frac{\beta}{(M^{4-1})M^{2-1}} + \frac{\gamma}{(M^{2-1})^3}$
- Simple nonflow with strong 2-particle correlations
  - Maximises nonflow
- χ^2/ndf = 0.7
- Behavior as expected
  - β, γ compatible with zero
  - ➔ Dominant contribution to nonflow from 6-particle correlator

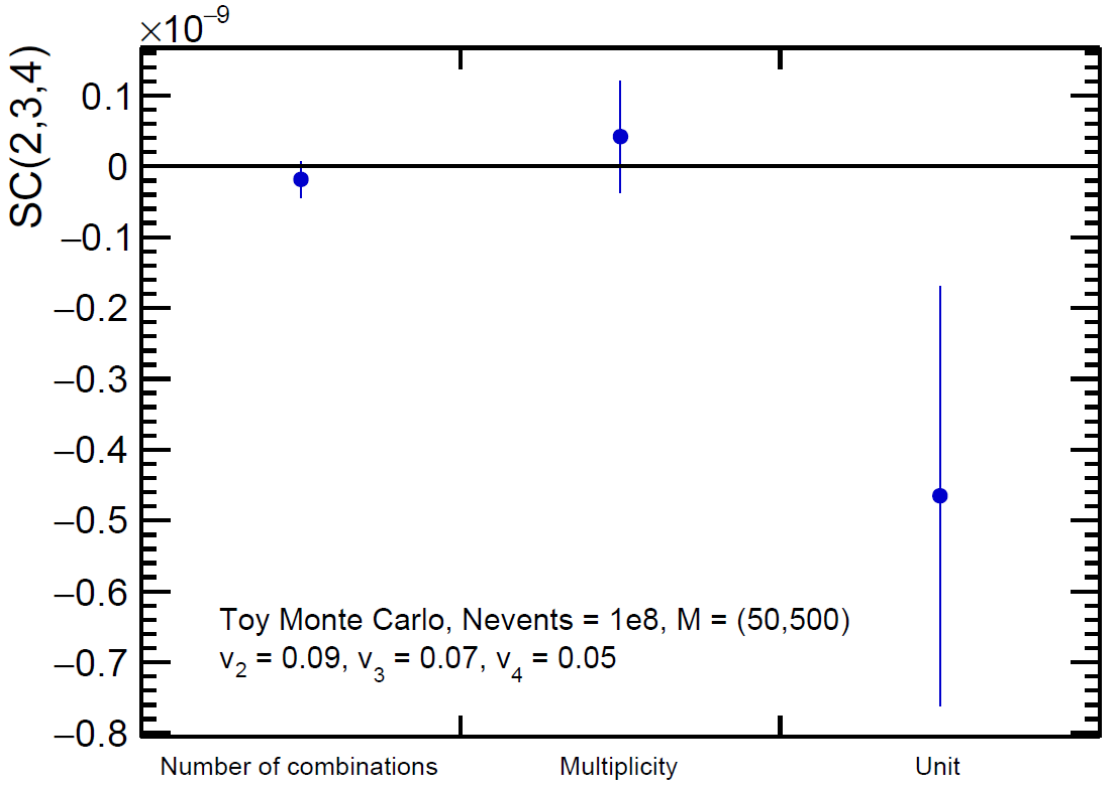
Nonflow scaling of a k-particle correlator:
$\delta_k \sim \frac{1}{M^{k-1}}$

7) Robustness against Nonflow?



8) Multiplicity Weights?

- Which weights to pass from $\langle \ldots \rangle$ to $\langle\langle \ldots \rangle\rangle$?
  - Number of combinations for a k-particle correlator: $\prod_{i=0}^{k-1}(M-i)$
  - Multiplicity of the event *M*
  - Unit weight
- *M* uniformly sampled in (50, 500)
- Constant harmonics
- ➔ Smallest statistical spread for number of combinations
  - Same conclusion as for SC(*m,n*)

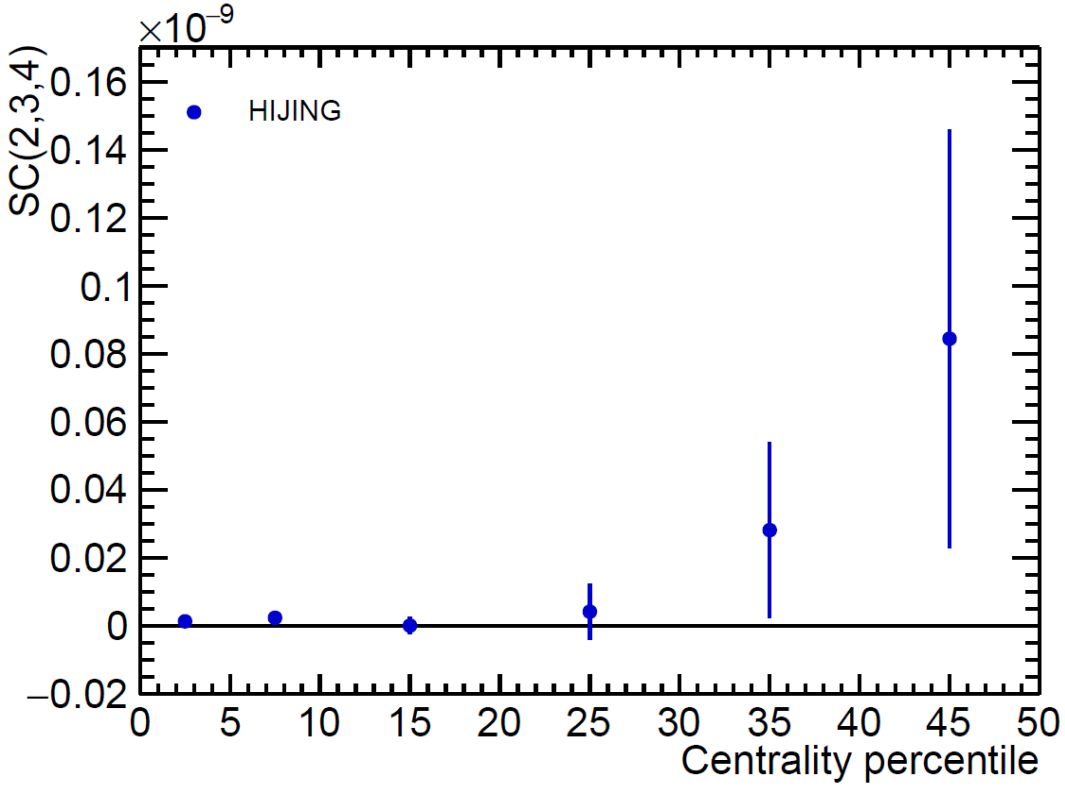


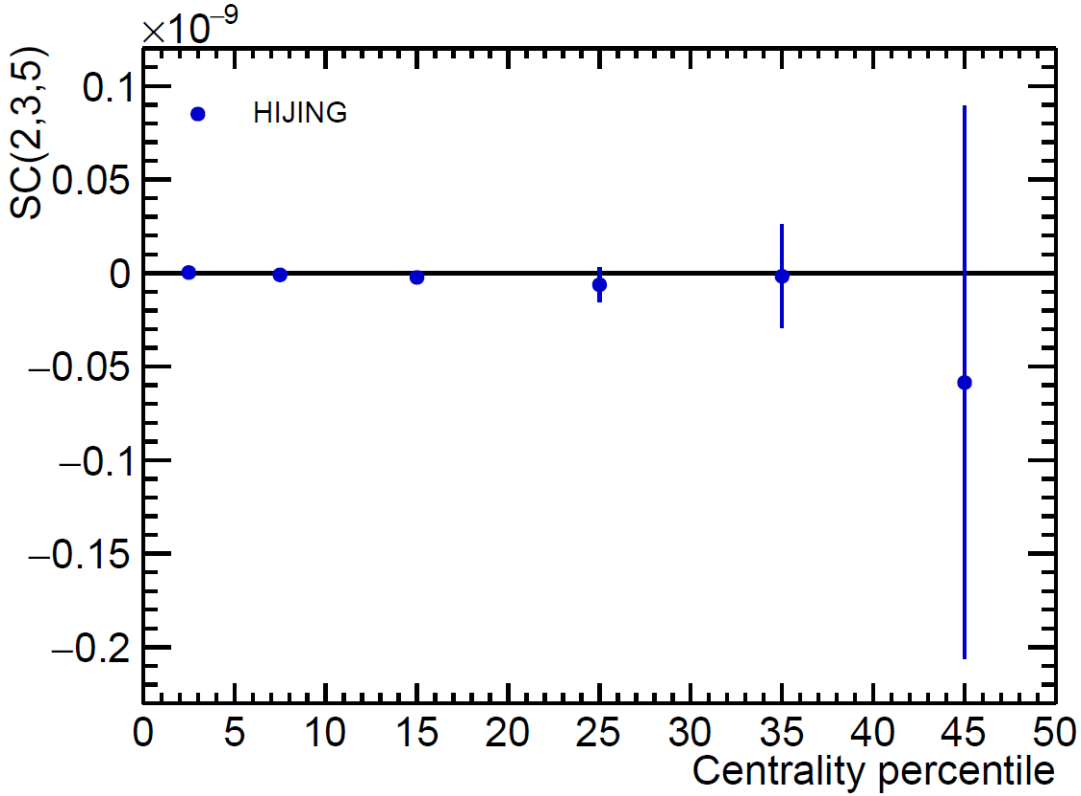
Realistic Monte Carlo studies

- Used two Monte Carlo models: HIJING and iEBE-VISHNU
- Heavy-Ion Jet INteraction Generator (HIJING):
  - Model to study particle and jet production in nuclear and heavy-ion collisions
  - Description of phenomena with correlations between few particles ➔ Only nonflow
  - Used to test the robustness of SC(k,l,m) against nonflow
- iEBE-VISHNU:
  - Heavy-ion collision event generator based on hydrodynamics calculations
  - Describes heavy-ion collision evolution
  - Contains both flow and nonflow, only flow used in this work
- For both, simulations of Pb-Pb collisions with $\sqrt{s} = 2.76$ TeV

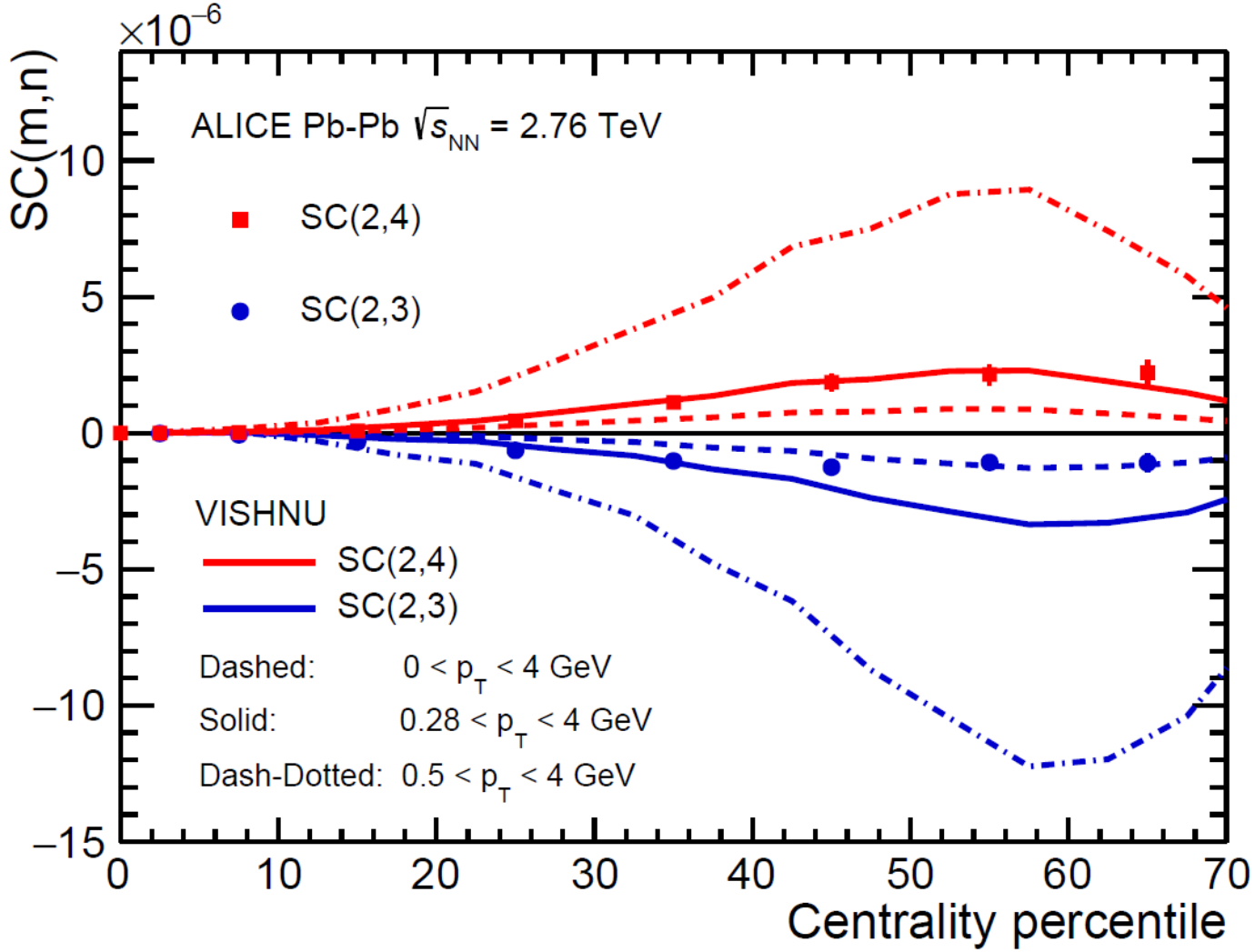
HIJING Simulations

- SC(k,l,m) compatible with zero for head−on and mid−central collisions ➔ Not sensitive to nonflow





Cross-Check of VISHNU with ALICE Data



- Comparison of ALICE 2010 Pb-Pb data with VISHNU simulations
- Sensitivity of SC on the lower limit of the p_T range
- $0.28 < p_T < 4$ GeV: better qualitative agreement with data
  - ALICE p_T range: $0.2 < p_T < 5$ GeV
- Dependence of SC(3,2) on η/s not studied here
  - Already studied in ALICE Collaboration, PRL 117, 182301 (2016)

Comparison between HIJING and VISHNU

- Values from HIJING smaller than the ones from VISHNU
- Results from VISHNU not a systematic bias of HIJING

Any nonzero results of SC in real data
➔ **Collective flow effects**