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Magnetized rotational neutron stars and the MR relations

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- Metric and Hartle equation
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Motivation

Two solar mass problem

2 neutron stars with around twice the solar mass.

$$\text{J1614-2230} \;:\; 1.97 \pm 0.04 M_{\odot}$$

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Nature **467**, (2010) 1081-1083

$$\text{J0348+0432} \;:\; 2.01 \pm 0.04 M_{\odot}$$

John Antoniadis *et al.*
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⇒ Such a very heavy neutron star gives a strong limit on the equation of state.

How can we explain $2M_\odot$ NS ?

We consider magnetic fields or/and rotation.

NS with strong magnetic field or/and rapid rotation may have such a large mass.

$$\mathrm{B} \sim 2 \times 10^{15}\mathrm{Gauss}$$

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APJ Supplement Series,212:6 (22pp) (2014)

$$\mathrm{Rotation} = 716\ \mathrm{Hz}$$

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About Radius

We can limit the radius from the observation.
The radii suggested by observational considerations.

Mass ($M_\odot$)	Radius (km)	method
0.86–2.42	> 7.6–10.4	Black body from surface Guillot *et al.* (2013)
1.2–1.7	< 9.0–13.2	Eddington limit Zamfir *et al.* (2012)
@ 1.4	> 6.6	The absorption line red shift Waki *et al.* (1984)
@ 1.4	$\lesssim$ 13.6	Gravitational wave Annala *et al.* (2018)

Formulation

Metric of slowly rotating neutron star considering axial deformation in GR.

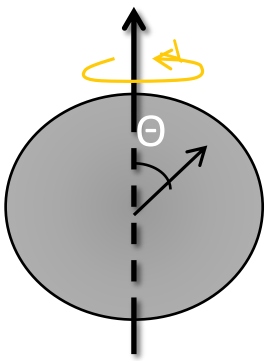
$$ds^2 = -e^{2\nu_0}\left[1 - 2h_0(r) + 2h_2(r)P_2(\cos\theta)\right]dt^2$$

$$+e^{2\lambda_0}\left\{1 + \frac{e^{2\lambda_0}}{r}\left[2m_0(r) + 2m_2(r)P_2(\cos\theta)\right]\right\}dr^2$$

$$+r^2\left[1 + 2k_2(r)P_2(\cos\theta)\right]\left\{d\theta^2 + \left[d\phi - \omega(r)\,dt\right]^2\sin^2\theta\right\}$$

ω : angular velocity

$P_2(\cos\theta)$: Legendre's polynomial of order 2



S. Chandrasekhar & John C. Miller,
Mon. Not. R. astr. Soc., 167, 63-79 (1974)

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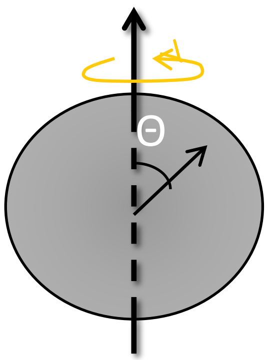

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Hartle Equations

To calculate the additional mass for slowly rotating neutron star.

$$\frac{1}{r^3}\frac{d}{dr}\left(r^4 j\frac{d\varpi}{dr}\right)+4\frac{dj}{dr}\varpi=0$$

$$-\frac{d}{dr}\delta P_0+\frac{1}{3}\frac{d}{dr}\left(r^2e^{-2\nu_0}\varpi^2\right)=m_0e^{4\lambda_0}\left(\frac{1}{r^2}+8\pi p_0\right)-\frac{1}{12}e^{2\lambda_0}r^3j^2\left(\frac{d\varpi}{dr}\right)^2+4\pi re^{2\lambda_0}\left(\varepsilon+p\right)\delta P_0$$

$$\frac{dm_0}{dr}=4\pi r^2\left(\varepsilon+p\right)\frac{d\varepsilon}{dp}\delta P_0+\frac{1}{12}r^4j^2\left(\frac{d\varpi}{dr}\right)^2-\frac{1}{3}r^3\varpi^2\frac{dj^2}{dr}$$

$$\frac{dv_2}{dr}=-2\frac{d\nu_0}{dr}h_2+\left(\frac{1}{r}+\frac{d\nu_0}{dr}\right)\left[\frac{1}{6}r^4j^2\left(\frac{d\varpi}{dr}\right)^2-\frac{1}{3}r^3\varpi^2\frac{dj^2}{dr}\right]$$

$$\frac{dh_2}{dr}=-\frac{2v_2}{r(r-2M)d\nu_0/dr}+\left\{-2\frac{d\nu_0}{dr}+\frac{r}{2(r-2M)d\nu_0/dr}\left[8\pi\left(\varepsilon+p\right)-\frac{4M}{r^3}\right]\right\}h_2$$
$$+\frac{1}{6}\left[r\frac{d\nu_0}{dr}-\frac{1}{2(r-2M)d\nu_0/dr}\right]r^3j^2\left(\frac{d\varpi}{dr}\right)^2-\frac{1}{3}\left[r\frac{d\nu_0}{dr}+\frac{1}{2(r-2M)d\nu_0/dr}\right]r^2\varpi^2\frac{dj^2}{dr}$$

S. Chandrasekhar & John C. Miller,
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Relativistic Mean Field (RMF) Theory

We use the following Lagrangian which includes interactions between baryon octet and σ, ω, ρ, σ*,and ϕ mesons.

$$
\begin{aligned}
\mathcal{L} = \sum_b \Big(& \overline{\psi}_b \left(i\gamma_\mu \partial^\mu - m_b + g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* - g_{\omega b}\gamma_\mu \omega^\mu - g_{\phi b}\gamma_\mu \phi^\mu - g_{\rho b}\gamma_\mu \boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu \right. \\
& \left. - q_b \gamma_\mu A^\mu - \kappa_b \sigma_{\mu\nu} F^{\mu\nu} \right) \psi_b \Big) \\
& + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) + \frac{1}{2}\left(\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}\right) \\
& + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu - \frac{1}{4}\Phi_{\mu\nu}\Phi^{\mu\nu} \\
& + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu - \frac{1}{4}\boldsymbol{P}^{\mu\nu}\cdot\boldsymbol{P}_{\mu\nu} \\
& - \frac{1}{3} b m_n \left(g_\sigma \sigma\right)^3 - \frac{1}{4} c \left(g_\sigma \sigma\right)^4 + \frac{1}{4!}\xi \left(g_\omega^2 \omega_\mu \omega^\mu\right)^2 + \Lambda_\omega \left(g_\omega^2 \omega_\mu \omega^\mu\right)\left(g_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu\right) \\
& + \sum_l \Big(\overline{\psi}_l \left(i\gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l \right) \psi_l \Big) \\
& - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}.
\end{aligned}
$$

Nuclear properties

★Bethe-Weizsäcker formula (1935)

Nuclear binding energy can be described roughly by a liquid-drop model.

Radius of nuclei : $R=r_0A^{1/3}$ r_0~0.15fm

$$B(A,Z) \equiv Zm_p + (A-Z)m_n - M(A,Z)$$

$$= a_{vol}A - a_{surf}A^{2/3} - a_{coul}\frac{Z^2}{A^{1/3}} - a_{sym}\frac{(N-Z)^2}{A} + \delta(A).$$

volume	surface	coulomb	symmetry energy	Pairing energy
$\propto \frac{4\pi}{3}R^3$	$\propto 4\pi R^2$	$\propto \frac{Q^2}{R}$	$\frac{1}{2}\left(\frac{\partial^2}{\partial t^2}\frac{\varepsilon}{\rho}\right)_{\rho=\rho_0}$ $t=(\rho_n-\rho_p)/\rho$	
16.2MeV	19.0MeV	0.76MeV	23.5MeV	1MeV

Nuclear properties

Binding energy per nucleon

$$B(A,Z)/A = a_{vol} = 16.2 \text{ MeV}$$

Nucleon number density

$$\rho_0 = \frac{a}{4\pi R^3/3} = 0.15\,\text{nucleon}/\text{fm}^3$$

Symmetry energy

$$a_{sym} = \frac{1}{2}\left(\frac{\partial^2}{\partial t^2}\frac{\varepsilon}{\rho}\right)_{\rho=\rho_0} \quad (t = (\rho_n - \rho_p)/\rho) \qquad 23.5\text{MeV}$$

Incompressibility

$$K = \left[k^2 \frac{d^2}{dk^2}\left(\frac{\varepsilon}{\rho}\right)\right]_{k=k_F} = 9\left[\rho^2 \frac{d^2}{d\rho^2}\left(\frac{\varepsilon}{\rho}\right)\right]_{\rho=\rho_0}$$

Symmetry-energy slope parameter

$$L = 3\rho_0 \left(\frac{dS}{d\rho}\right)_{\rho_0} \qquad a_{sym} = S(\rho_0)$$

incompressibility of symmetry energy

$$K_{sym} = 9\rho_0^2 \left(\frac{d^2S}{d\rho^2}\right)_{\rho_0}$$

Properties of various EoSs

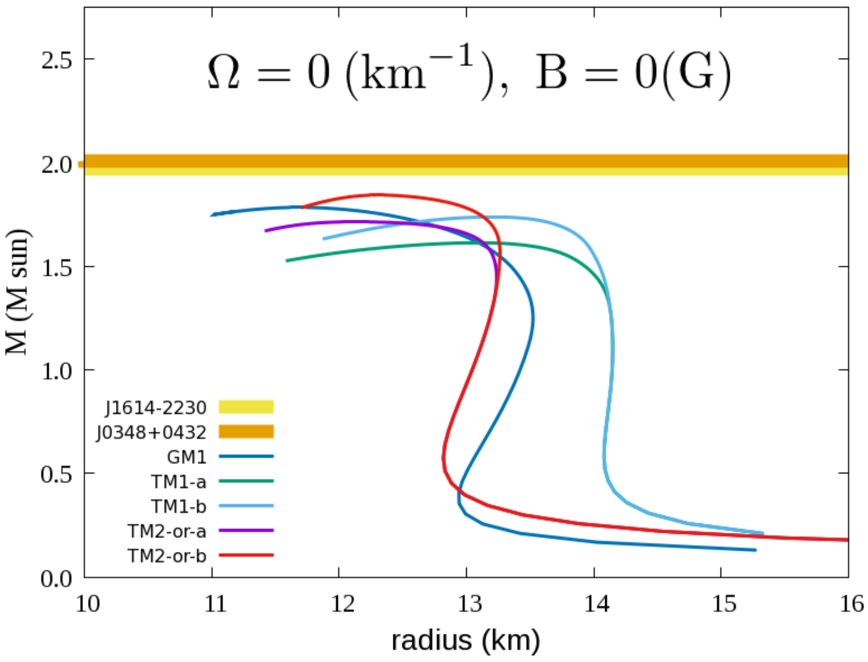
EoS	B/A MeV	ρ_0 fm^{-3}	a_{sym} MeV	K MeV	L MeV	K_{sym} fm^{-3}
GM1(*1)	16.3	0.153	32.5	300	94.4	18.1
TM1-a(*2)	16.3	0.146	36.9	281.2	111.2	33.8
TM1-b(*2)	16.3	0.146	36.9	281.2	111.2	33.8
TM2-ωρ-a(*2)	16.4	0.146	32.1	281.7	54.8	-70.5
TM2-ωρ-b(*2)	16.4	0.146	32.1	281.7	54.8	-70.5

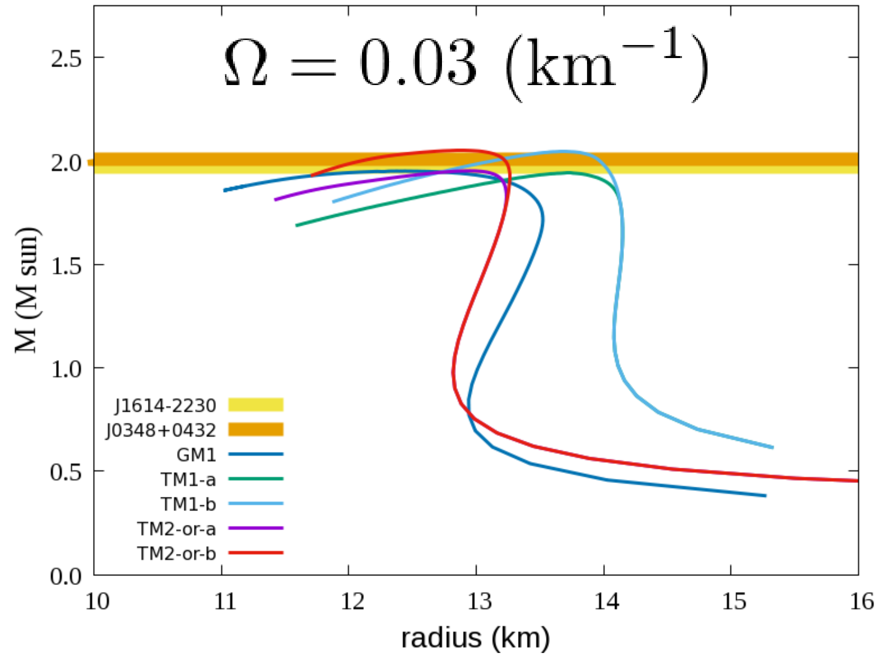
(*1)N.Glendenning & S.Moszkowski Phy.Rev.Letter, vol.67, Num.18(1991)

(*2) M.Fortin et al., Physical Review C95, 065803 (2017)

Results and Summary

Comparison of rotating NS masses (5 EoSs)





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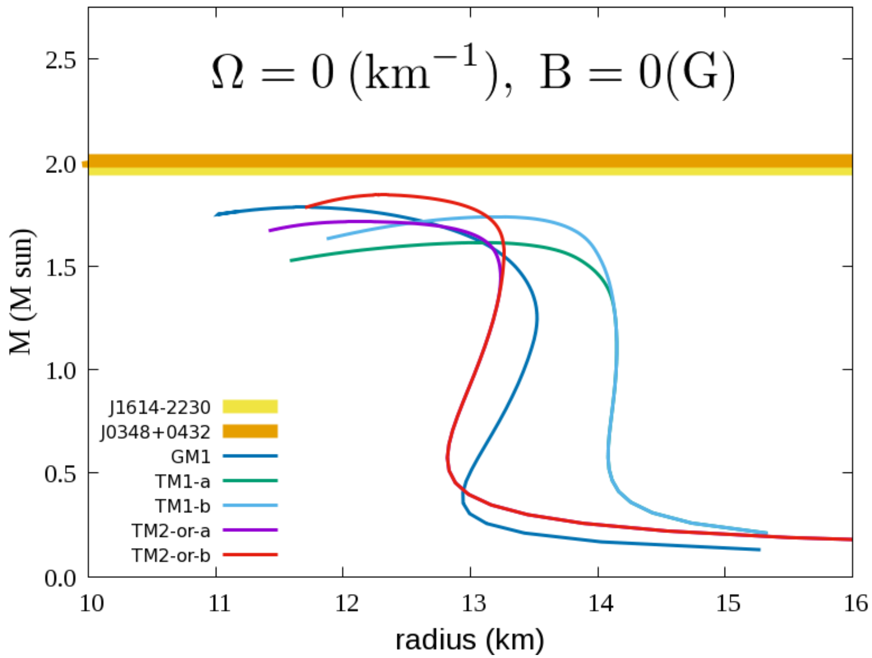

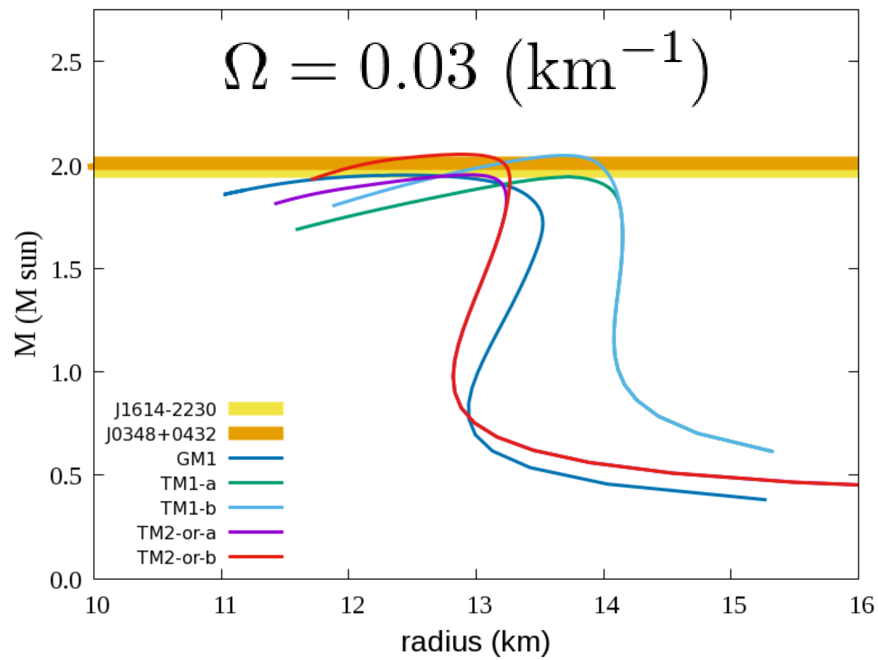

TM1-b and TM2-ωρ-b EoSs give over twice the solar mass.

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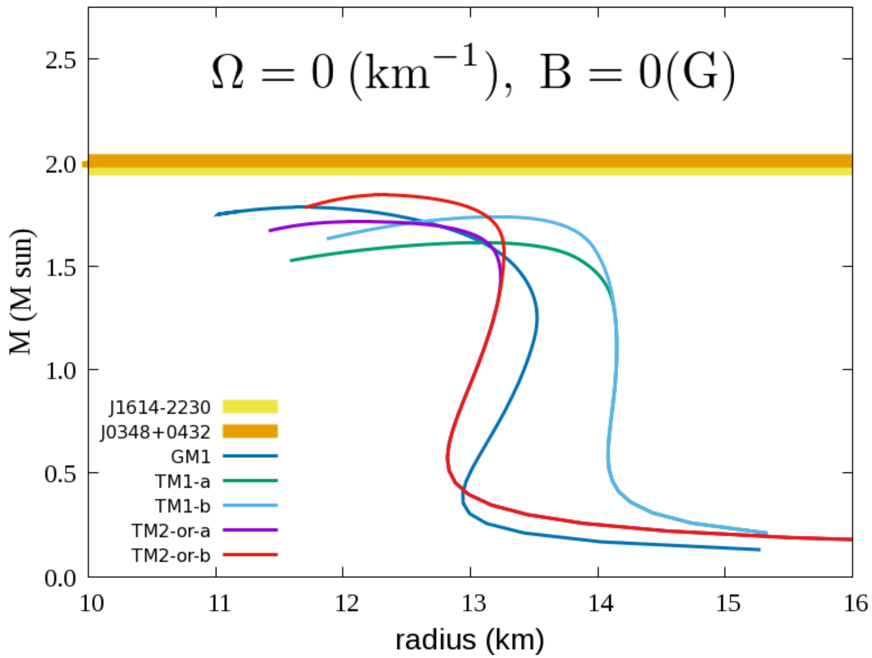
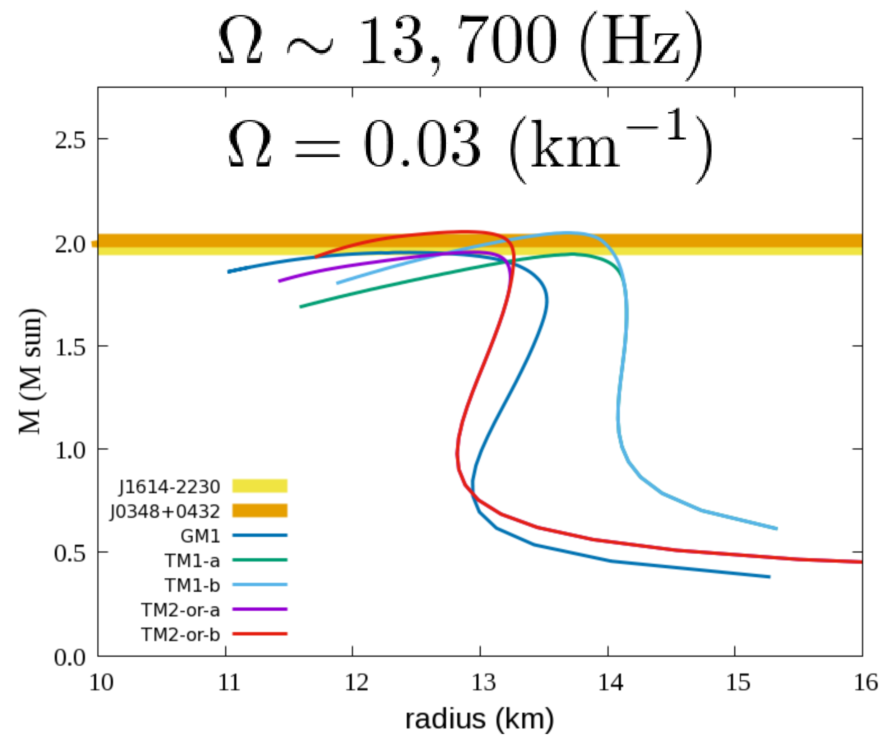
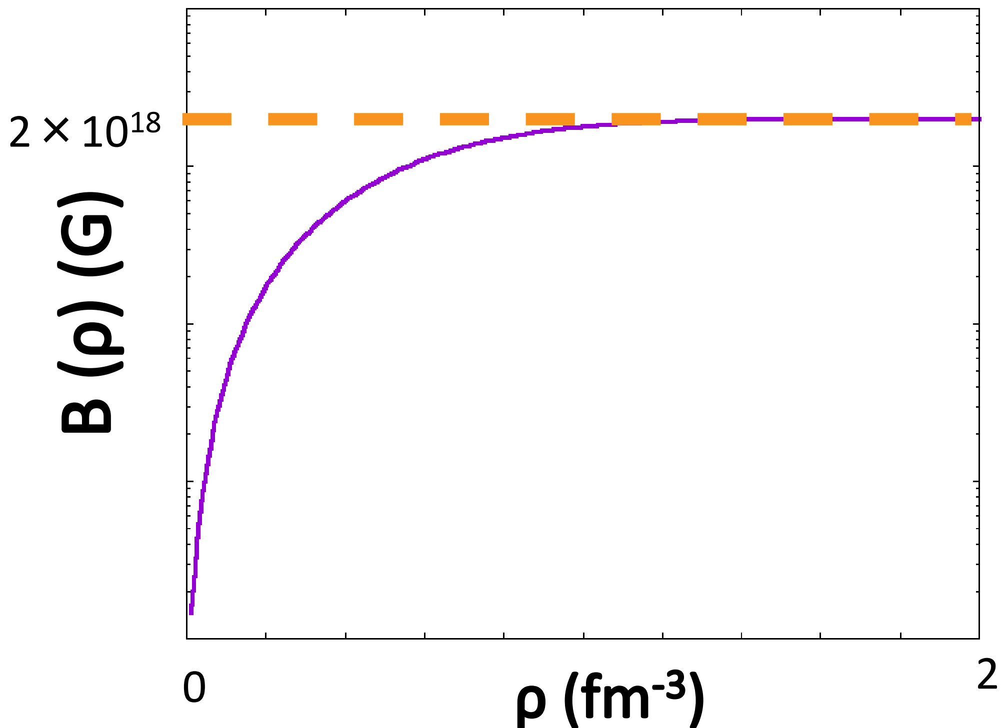




TM1-b and TM2-ωρ-b EoSs give over twice the solar mass.

Magnetic field B(ρ)

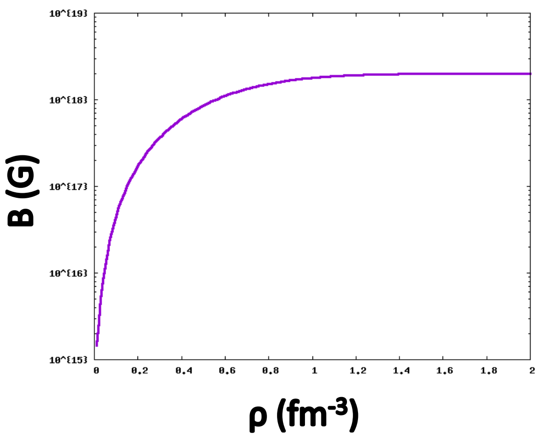
For magnetic field, we used following $B(\rho)$
(Spherically Symmetric Magnetic Pressure)



Magnetic field B(ρ)

For magnetic field, we used following $B(\rho)$
(Spherically Symmetric Magnetic Pressure)

$$B(\rho) = B_s + B_0 \left[1 - \exp\left\{ -\alpha \left(\frac{\rho}{\rho_0} \right)^{\gamma} \right\} \right]$$



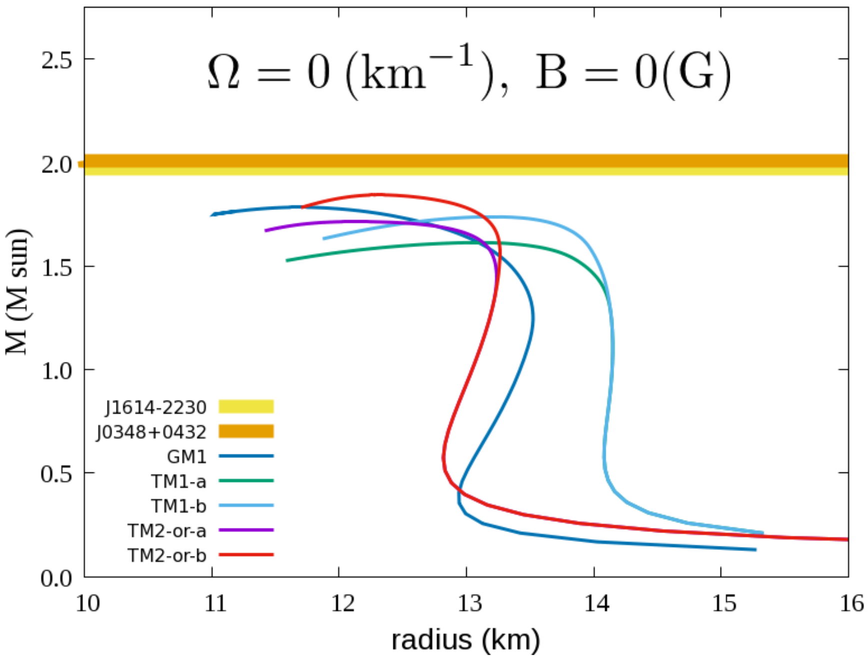
$$B_s = 1 \times 10^{15} (G) \qquad \alpha = 0.05, \gamma = 2$$

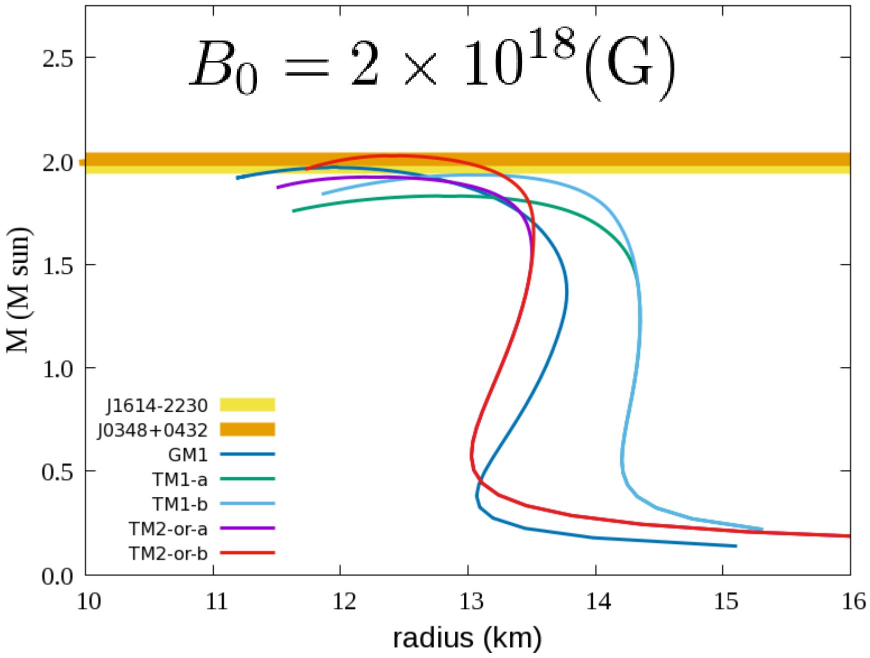
$$B_0 = 2 \times 10^{18} (\mathrm{G})$$

ρ : number density, ρ_0 : normal nuclear density

R.H.Castro , L.B.Castro & D.P.Menezes,
Phy. Rev. C 89, 015805 (2014)

Comparison of magnetized NS masses (5 EoSs)





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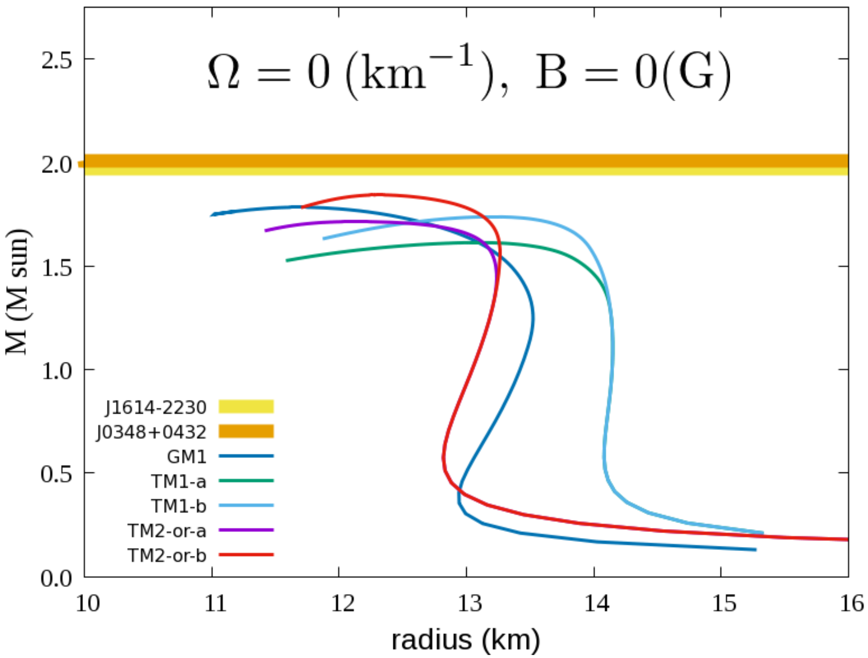
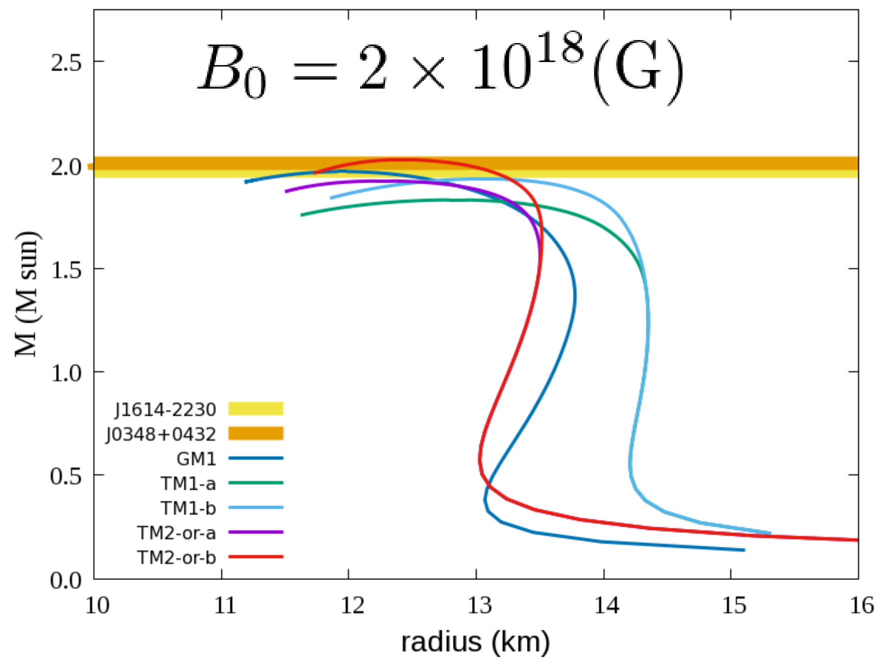
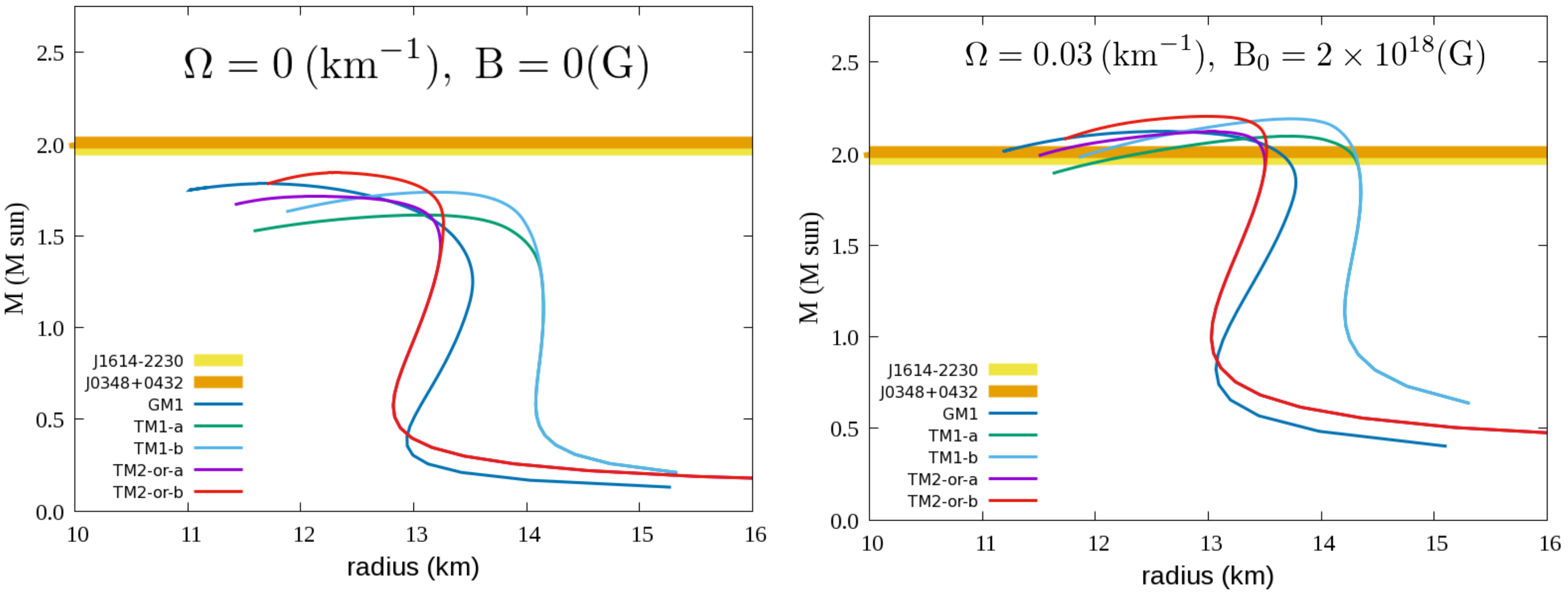




Only TM2-ωρ-b EoS gives over twice the solar mass.

Comparison of rotating and magnetized NS (5 EoSs)



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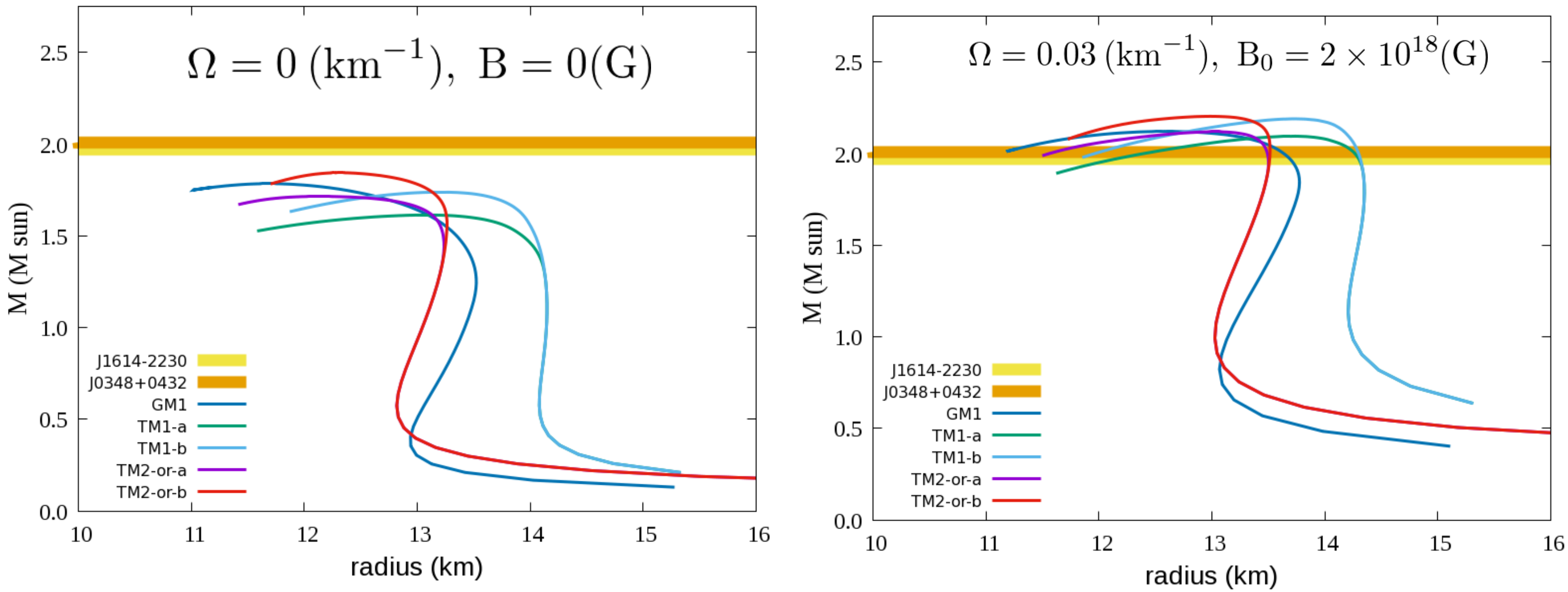

All 5 EoSs give over twice the solar mass.

Radius for $1.4M_{\odot}$ NS

EoS	R km	$R_{(rot)}$ km	$R_{(mag)}$ km	$R_{(rot\&mag)}$ km
GM1	13.46	13.34	13.77	13.49
TM1-a	14.07	14.12	14.33	14.26
TM1-b	14.10	14.12	14.33	14.26
TM2-ωρ-a	13.23	13.02	13.47	13.21
TM2-ωρ-b	13.24	13.02	13.47	13.21

Observation
$6.6 < R \leqq 13.6$ km

Waki *et al.* (1984)

Annala *et al.* (2018)

$\Omega = 0.03\,\mathrm{km}^{-1}$ $B_0 = 2 \times 10^{18}\,\mathrm{G}$

Radius for $1.4M_\odot$ NS

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$\Omega=0.03\text{km}^{-1}$ $B_0=2\times10^{18}$G

Observation
$6.6 < R \leqq 13.6$ km

Approximately
GM1 &
TM2-ωρ-a &
TM2-ωρ-b
are in the range.

Radius for 1.4$M_\odot$ NS

EoS	R km	$R_{(rot)}$ km	$R_{(mag)}$ km	[illegible]
GM1	13.46	13.34	13.77	[illegible]
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TM1-b	14.10	14.12	14.33	[illegible]
TM2-ωρ-a	13.23	13.02	13.47	13.21
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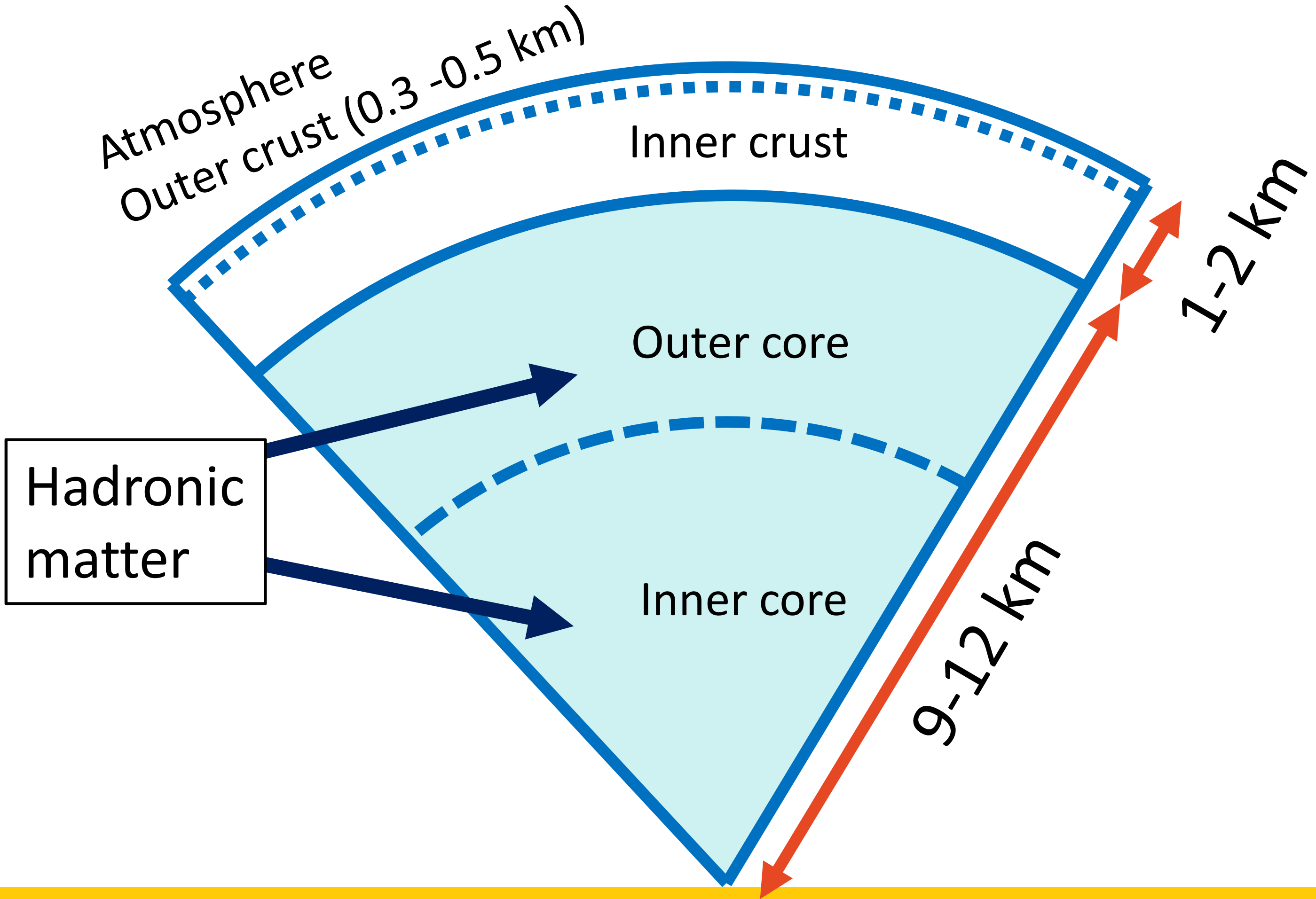
Ω=0.03km^{-1} B_0=2×10^{18}G

EoS	L MeV
GM1(*1)	94.4
TM1-a(*2)	111.2
TM1-b(*2)	111.2
TM2-ωρ-a(*2)	54.8
TM2-ωρ-b(*2)	54.8

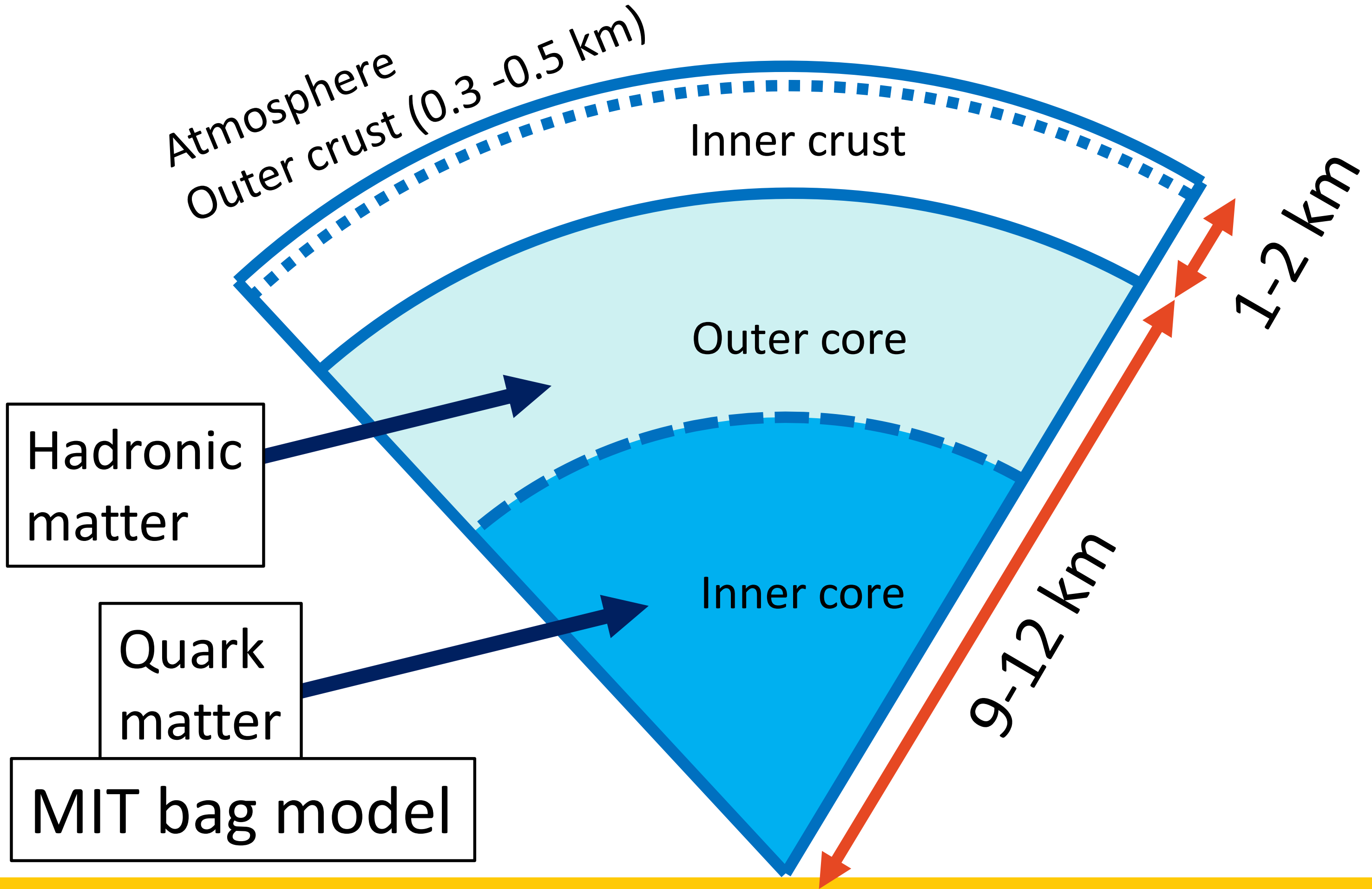
m

GM1 &
TM2-ωρ-a &
TM2-ωρ-b
are in the range.

Neutron star

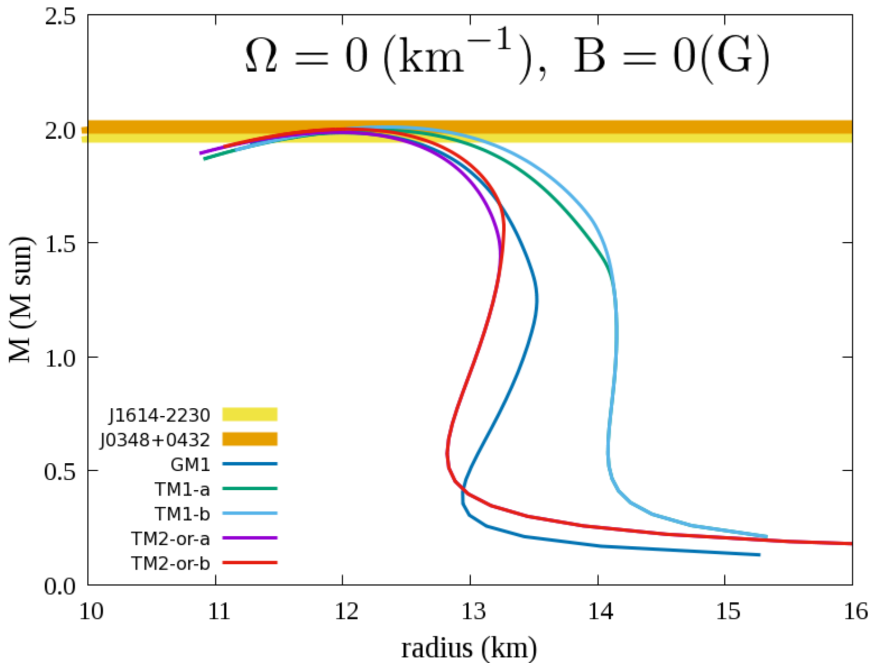


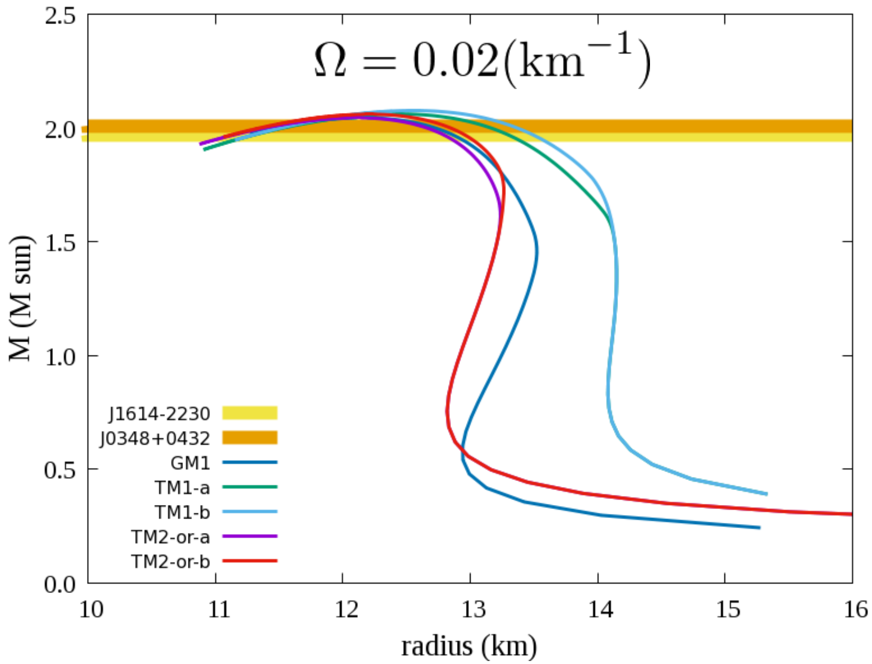
Hybrid star



Atmosphere
Outer crust (0.3 -0.5 km)
Inner crust
Outer core
Inner core
Hadronic matter
Quark matter
MIT bag model
1-2 km
9-12 km

Comparison of Hybrid star with rotation(5 EoSs)





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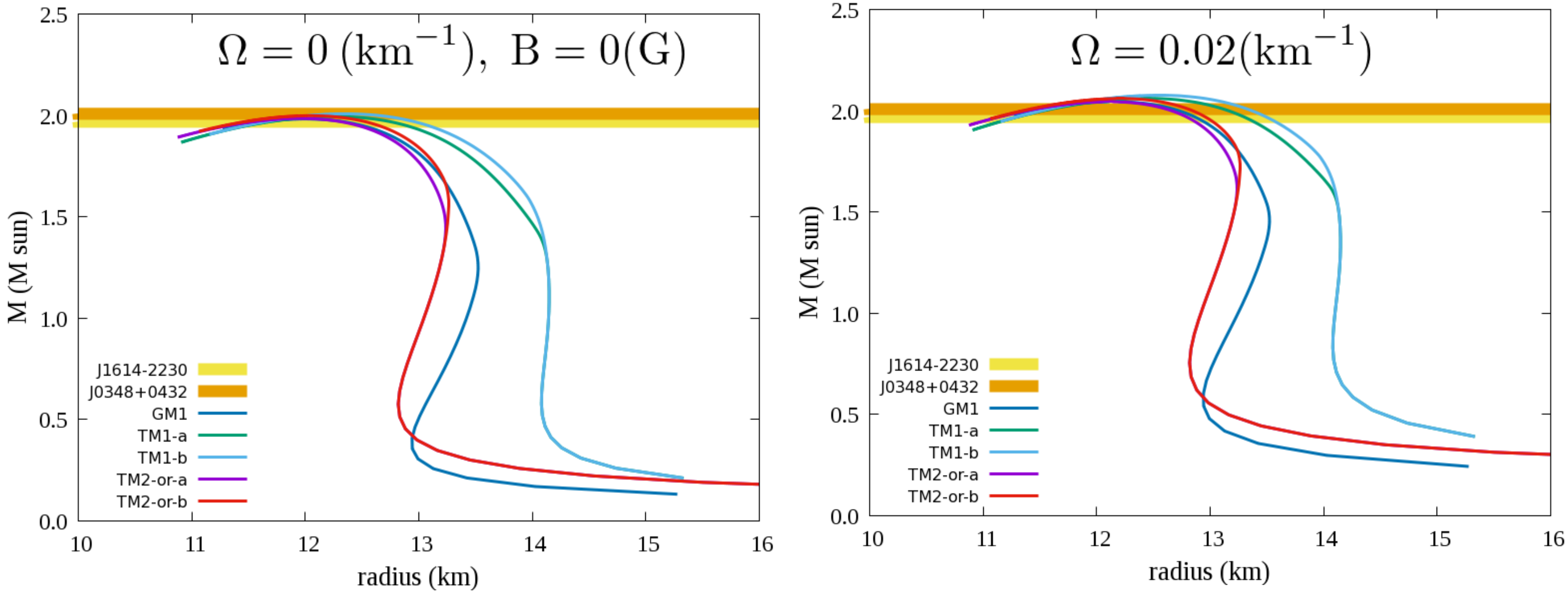

All 5 EoSs give over twice the solar mass.

Summary

- ✓ We calculated the mass-radius relations for magnetized and rotating neutron stars using various kinds of EoSs.
- ✓ We obtained neutron stars with masses more than 2 $M_{\odot}$ both with strong magnetic fields and in rapid rotations for 5 hadronic EoSs.
- ✓ From the radius, GM1, TM2-ωρ-a and TM2-ωρ-b EoSs are in the range of observation.
- ✓ Hybrid star with rapid rotation give a mass over twice the solar mass for 5 hadronic EoSs.

Future work

- To calculate MR relations for hybrid stars (mixture of quarks and hadrons) under the circumstance of rotation and magnetic fields.

- If the rotational axis and the deformation axis are different, gravitational waves might occur (wobbling motion). We are planning to look into that.

Some part of our work will be published in PTEP soon.