Structure Contributions to the Deuterium Lamb Shift in Chiral EFT

INPC 2019

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August 1, 2019

Overview

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The proton radius puzzle

$\sim 5.6\sigma$ discrepancy between measurements of the proton radius from electronic and muonic hydrogen[1].

$$r_p = 0.8751(61)\mathrm{fm}, \quad \text{(electronic hydrogen)}$$
$$r_p = 0.84087(39)\mathrm{fm}, \quad \text{(muonic hydrogen)}$$

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[2] Pohl, R. *et al.*, Science **353** 669 (2016)

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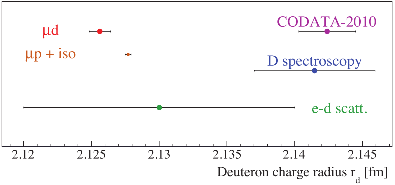
A similar problem exists for the deuteron[2] ($\sim 3.5\sigma$) :



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Measuring the charge radius

Correction to $-\frac{\alpha}{r}$ Coulomb potential:

$$\delta V(r) = -\alpha \int \mathrm{d}^3\xi \, \rho(\vec{\xi}) \left(\frac{1}{|\vec{r} - \vec{\xi}|} - \frac{1}{r} \right)$$

Leading time-independent perturbation theory result

$$\Delta E = \delta_{l0} \frac{2\mu^3\alpha^4}{3n^3} \langle r_E^2 \rangle, \quad \langle r_E^2 \rangle = -6 \left. \frac{\mathrm{d}G_E}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

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Can obtain charge radius from S-P splitting, *if other contributions are understood*

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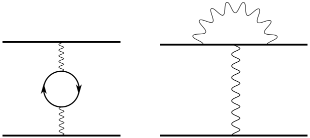
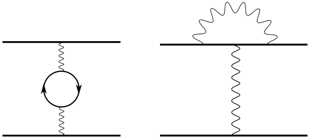
The Lamb shift

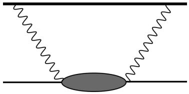
- S-P splitting in hydrogen dominated by QED effects
- Well-understood theoretically



The Lamb shift

- S-P splitting in hydrogen dominated by QED effects
- Well-understood theoretically
- Structure effects are significant in deuterium
- Leading source of theoretical uncertainty in muonic deuterium spectroscopy





$$\Delta E_{nl} = \frac{2\mu^3\alpha^4}{3n^3}\langle r_E^2\rangle + \delta_{QED} + \delta_{TPE}$$

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Motivation

Other methods exist for calculating two-photon exchange, e.g. dispersion relations[1] or from a multipole expansion of the deuteron polarisability [2] $\rightarrow$ why take another approach?

[1] Carlson, C.E. *et al.*, Phys. Rev. A, 89 (2014) p. 022504

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Motivation

Other methods exist for calculating two-photon exchange, e.g. dispersion relations[1] or from a multipole expansion of the deuteron polarisability [2] $\rightarrow$ why take another approach?

- Consistent treatment of nuclear and nucleon-level effects in same framework
- Simple formalism and treatment of nuclear physics
- Systematic calculation from chiral EFT (if a chiral potential is used)

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Virtual Compton scattering

All hadronic contributions to two-photon exchange are contained in the virtual spin-averaged Compton tensor

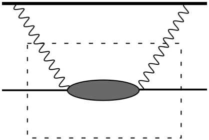

Figure: Two-photon exchange

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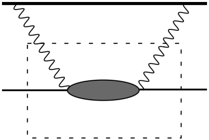

Figure: Two-photon exchange

Compton tensor

$$T^{\mu\nu} = \frac{\mathrm{i}}{3}\sum_{\lambda}\int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}q\cdot\xi}\langle d,\lambda|\, T\left\{j^{\mu}(\xi)j^{\nu}(0)\right\}|d,\lambda\rangle.$$

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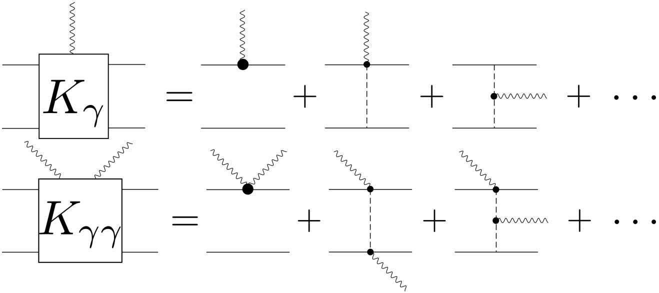

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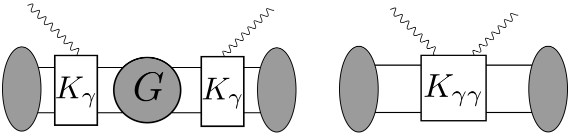
Method of calculation

- Solve Schrödinger equation with N-N potential for wave function and full Green's function
- Calculate electromagnetic N-N current operators in chiral EFT
- Stitch together to give Compton amplitude



Rescattering contribution

The rescattering contribution to the Compton tensor (intermediate $2N$-state) is given by

$$T_{\lambda_f\lambda_i} = \sum_C \left\{ \frac{\langle d_f;\gamma_f|\hat{H}_{\lambda_f}|C\rangle\langle C|\hat{H}_{\lambda_i}|d_i;\gamma_i\rangle}{\omega + E_i - E_C + \mathrm{i}\varepsilon} \right.$$
$$\left. \frac{\langle d_f;\gamma_f|\hat{H}_{\lambda_i}|C;\gamma_f;\gamma_i\rangle\langle C;\gamma_f;\gamma_i|\hat{H}_{\lambda_f}|d_i;\gamma_i\rangle}{-\omega - E_0 - E_C + \mathrm{i}\varepsilon} \right\},$$

where the interaction is given by

$$\hat{H}_\lambda = -\int \mathrm{d}^3\xi \hat{\varepsilon}_\lambda \cdot \mathbf{j}(\vec{\xi})\, \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{\xi}}.$$

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For long wavelengths, the photon field may be written as a gradient $\mathbf{A} \approx \vec{\nabla}\phi$, giving an interaction

$$\hat{H}_{\lambda}^{\phi} = \mathrm{i} \int \mathrm{d}^3\xi \, \left[\rho(\vec{\xi}), \hat{H}\right] \phi_{\lambda}(\vec{\xi}).$$

- At low-energies, the amplitude depends only on the **charge density** of the nucleus - *Siegert's theorem*[1].
- Corrections from residual field $\mathbf{A} - \vec{\nabla}\phi$ included separately

[1] Siegert, A. J. F., Phys. Rev. 52, 737 (1937)

Two-photon exchange

Our aim is to calculate the TPE diagram:

Lamb shift integral

$$\Delta E_{nl} = -\frac{(4\pi\alpha)^2}{m} |\phi(0)_{nl}|^2 \frac{1}{\mathrm{i}} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} D^{\mu\lambda}(q) D^{\nu\sigma}(-q) t_{\mu\nu}(p,q) T_{\lambda\sigma}(q),$$

with the leptonic tensor $t^{\mu\nu}$ defined as

$$t_{\mu\nu}(p,q) = \frac{p^\mu(p^\nu - q^\nu) + (p^\mu - q^\mu)p^\nu + p \cdot q g^{\mu\nu}}{(p-q)^2 - m_\mu^2 + \mathrm{i}\varepsilon}$$

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In Coulomb gauge, from crossing symmetry we obtain

$$\Delta E_{nl} = -(4\pi\alpha)^2 |\phi(0)_{nl}|^2 \frac{1}{\mathrm{i}} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{2m_l^2}{q^4 - 4m_l^2 q_0^2 + \mathrm{i}\varepsilon} \times \left[\frac{1}{|\vec{q}|^2} T_L(q_0, |\vec{q}|) + \frac{q_0^2}{q^4} T_T(q_0, |\vec{q}|) \right],$$

where the Compton tensor has separated into transverse and longitudinal components

$$T_L \equiv T_{00}, \quad T_T \equiv \left(\delta^{ij} - \frac{q^i q^j}{|\vec{q}|^2} \right) T^{ij}$$

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Elastic and inelastic

We calculate the full Compton tensor, including ground-state contributions

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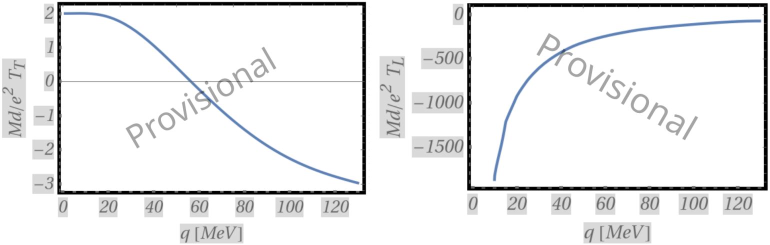
$$G_C(Q^2) = 1 - \frac{\langle r_E^2 \rangle}{6} Q^2 + \dots$$

It is therefore necessary to subtract elastic contributions up to $\mathcal{O}(Q^2)$ from the Compton tensor
$\rightarrow$ separately calculate ground-state contribution, and subtract from amplitude



(a) Transverse amplitude (b) Longitudinal amplitude

(amplitudes computed at $\omega = \sqrt{m_\mu^2 + q^2} - m_\mu$)

Summary

- The largest theoretical uncertainty in the Lamb shift in muonic deuterium is due to two-photon exchange
- Using an effective theory approach, we can incorporate single and two-nucleon effects in a common framework to calculate the deuteron virtual Compton tensor
- From this we hope to calculate the full inelastic contribution to the muonic deuterium lamb shift

Summary

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Thank you!

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